Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

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INTERNSHIP REPORT

VALUATION OF NON-LIFE CLAIMS PROVISION AND THE CAPITAL REQUIREMENT FOR RE-SERVE RISK

ANA CATARINA TEIXEIRA CASTRO

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ABSTRACT

A few selected methods to assess claims provision are applied including the corresponding variability measures, considering the Solvency II valuation principles. Besides the literature on claims reserving is very much diversified, we decided to focus on some of the methods more commonly used and explored. Being them: the Thomas Mack's model, the Bühlmann-Straub model and the Over-Dispersed Poisson Generalized Linear Model. The calculation of the reserve risk capital charge was also focused by implementing and comparing different approaches, being them: the standard formula with and without undertaking specific parameters and a partial internal model. Lastly, two different simplifications to calculate the risk margin were pursued. Such approaches are based on the cost-of-capital method and refer to the first and second simplifications of the hierarchy of simplified methods to calculate the risk margin set out in EIOPA guidelines on the valuation of technical provisions. In the end, a case study involving the methodologies implemented and some sensitivity analysis were applied to a sample of data for Motor Vehicle Liability line of business.

Keywords: Solvency II; Claims Provision; Reserve Risk; Solvency Capital Requirement; Risk Margin.

SUMÁRIO

Foram selecionados alguns métodos para calcular as provisões para sinistros, assim como as correspondentes medidas de variabilidade, tendo em consideração os princípios de avaliação de Solvência II. Para além de a literatura existente sobre provisões para sinistros ser bastante diversificada, decidimos focarmo-nos apenas nalguns dos métodos mais usados e explorados. Sendo eles: o modelo de Thomas Mack, o modelo de Bühlmann-Straub e o modelo linear generalizado com distribuição de sobre-dispersão de Poisson. O cálculo do requisito de capital do risco de provisões também foi efetuado através da implementação e comparação de diferentes abordagens, sendo eles: a fórmula padrão com e sem utilização dos parâmetros específicos da empresa e um modelo interno parcial. Por último, foram ainda implementadas duas simplificações para calcular a margem de risco. Tais métodos são baseadas na abordagem de custo de capital e referem-se às duas primeiras simplificações da hierarquia dos modelos simplificados para calcular a margem de risco estabelecidos nas orientações da EIOPA sobre a avaliação das provisões técnicas. No final, foi apresentado um caso de estudo onde se aplicou as metodologias implementadas e algumas análises de sensibilidade a uma amostra de dados para a linha de negócio Automóvel – Responsabilidade Civil.

Palavras-chave: Solvência II; Provisão para Sinistros; Risco de Provisões; Requisito de Capital de Solvência; Margem de Risco.

ACKNOWLEDGEMENTS

I would like to thank my supervisors Ana Rita Ramos and Hugo Borginho for the support and commitment.
I also thank Professor Walther Neuhaus who gave me extremely useful advices.
A special thanks to my family and Diogo Sá for their kindness, patience and incredible support.

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ABBREVIATIONS

ASF – Autoridade de Supervisão de Seguros e Fundos de Pensões

BI – Bodily-Injury

BE – Best Estimate

BS – Bühlmann-Straub

CDR – Claims Development Result

EIOPA – European Insurance and Occupational Pensions Authority

EU – European Union

GLM – Generalized Linear Model

HRG – Homogeneous Risk Groups

LoB – Line of Business

MD – Material Damages

MSEP – Mean Squared Error of Prediction

MW – Merz and Wüthrich

NSLT – Non-similar to life techniques

ODP – Over-Dispersed Poisson

RFR – Risk Free Rates

RM – Risk Margin

SCR – Solvency Capital Requirement

USP – Undertaking Specific Parameter(s)

VBA – Visual Basic for Applications

1. INTRODUCTION

This report aims to present the results of a six-month curricular internship at "Autoridade de Supervisão de Seguros e Fundos de Pensões" (ASF), working with the Solvency and Risk Analysis team. Its goal was to implement different methodologies towards the assessment of Non-Life Underwriting Risk, more specifically, the Reserve Risk, and the calculation of claims provision according to Solvency II principles. In order to illustrate some of the methodologies studied and implemented a case study on Motor vehicle liability insurance Line of Business (LoB) was developed and presented in this report.

One of the key decisions in the management of an insurance undertaking is the establishment of adequate technical provisions in order to ensure compliance with the insurer's obligations. Regarding the Non-life technical provisions, which are split into two different components - Claims Provisions and Premium Provisions – only the first component has been studied, i.e. the provisions for past claims (reported but not settled plus the incurred but not yet reported to the insurer). This item is composed by the sum of a best estimate and a risk margin. The best estimate value corresponds to the probability-weighted average of all future cash-flows, until run-off, taking account of the time value of money. In this work, we intend to present some well-known stochastic models to obtain an estimative for the best estimate. The Risk Margin represents the return required by a third party to accept a hypothetical transfer of the portfolio of insurance liabilities and it is based on the projection of the cost of capital for establishing the future regulatory capital requirements until the run-off of the liabilities transferred. The Solvency Capital Requirement (SCR) is another important element in the insurer's balance sheet and represents the extra capital needed to absorb unexpected losses. The SCR is divided into several risk sub-modules and the reserve risk is one of them. This risk can be defined as the risk that the amount of current reserves are in reality insufficient to cover the run-off position over a 12 month time horizon. To estimate this capital charge and consequently the component of the risk margin associated to this particular risk, a set of methods was selected and then applied to the Motor Vehicle LoB.

During the internship we were focused in one hand (a) on the improvement of some tools used internally by ASF for assessing the claims provision by implementing several methods as well as adapting the methods already applied towards the Solvency II regime. On the other hand (b) the calculation of the reserve risk capital charge, which is part of the SCR, was analyzed envisaging its computation using either the standard formula (including the application of the standardized methods to estimate undertaking specific parameters to produce the standard deviation) or a partial internal model. Once this stage was concluded, (c) different approaches to calculate the risk margin were pursued.

In order to estimate the outstanding claims provision, the Bühlmann-Straub model, the Mack's model and the Over-Dispersed Poisson Generalized Linear Model were implemented and updated from Solvency I to Solvency II valuation principles such as the introduction of the discount effect required by the current regime. For the reserve risk capital charge calculation, besides the standard approaches, an alternative methodology was proposed, which can be used as a partial internal model, based on the bootstrap technique. This methodology simulates the next one-year expected cumulative payments and used them to measure the uncertainty of claims reserves over a one-year time horizon. To project the future SCRs needed for the risk margin calculation two simplifications methods were presented. One method estimates the future required capital based on the ratio between the best estimate at that future year and the best estimate at the valuation date. The other one calculates the future SCRs using the mean square error of the claims development result for future years.

This document will start, in chapter 2, with a brief overview of the subject, including the Solvency II regime, the technical provisions components and the SCR. Chapter 3 presents the three statistical models selected to estimate the claims provision and their respective variability measures. In Chapter 4 it is introduced the concept of SCR and in particular the reserve risk as well as how it can be assessed through some different approaches. Chapter 5 includes the definition of the risk margin and how it can be calculated. Chapter 6 shows the results obtained by the different proposed methodologies presented in the previous chapters for the claims provision, SCR and risk margin. It also presents some sensitivity analysis performed by applying those methods. Such results are analyzed and compared. Finally, Chapter 7 concludes this report by summarizing the main findings.

2. SOLVENCY II OVERVIEW

The Solvency II regime has entered into force in January 1st 2016. It reviews, in depth, the prudential regime for insurance and reinsurance undertakings in the European Union (EU).
The legislative structure of this new regime comprises the following legal texts:

- Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009: lays down the general principles of the regime (level 1);
- Commission Delegated Regulation (EU) 2015/35 of 10 October 2014: implementing measures of the Solvency II Directive (level 2);
- Implementing Technical Standards and Regulatory Technical Standards: further mandatory detailed rules (level 2.5);
- EIOPA Guidelines: to ensure the convergence of practices within the EU (level3).

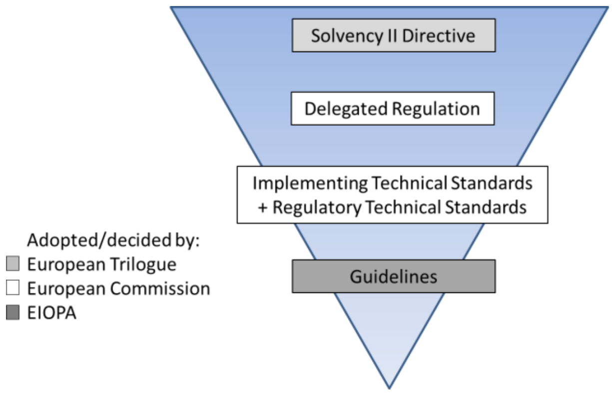


Figure 1: Legislative structure of Solvency II regime

The Solvency II key objectives were to enhance the policyholder's protection as well as the market discipline and transparency; deepen the single EU insurance market and promote a risk-based culture. Solvency II is divided into three areas, known as *pillars*, which aims a global and integrated view of risks:

Pilar I – quantitative requirements: the main objective is to provide a common valuation and risk measurement framework for all insurers in the EU;

Pillar II – qualitative requirements: the main objective is to set common rules to ensure a sound and prudent business management by the insurers;

Pillar III – reporting and disclosure of information: the main objective is to enforce the transparency and also to define a common reporting framework to supervisors.

Within the quantitative requirements, the technical provisions item stands out as the first level of policyholders' protection. The estimation of this item is an essential part of actuarial work in general insurance, since it means quantifying the sufficient amount to cover all the liabilities which were assumed towards policyholders and beneficiaries. According to article 76 of the Solvency II Directive, technical provisions should correspond to the amount that a (re)insurance undertaking will need to pay in order to immediately transfer the liabilities portfolio to other insurance or reinsurance undertaking in an arm's length transaction. Because of its relevance in the economic balance sheet, it must be calculated in a prudent, accurate and objective manner. The rules relating to the calculation of this item, including data quality, are presented in the chapter III of the Delegated Regulation.

Solvency II technical provisions are defined as the sum of a Best Estimate (BE) and a Risk Margin (RM). The first element is described as the probability-weighted average of the expected present value of all future cash-flows until run-off. The description of the cash flows that should be projected in the scope of the BE calculation can be found in the Chapter III, subsection 3, article 28 of the Delegated Regulation. For the purpose of discounting the basic risk-free interest rate term structure for the currency where the liabilities are denominated should be used. Such rates are published monthly by EIOPA. The Risk Margin represents the cost of risk, i.e. the return required by a third party to accept a hypothetical transfer of the portfolio of insurance liabilities in addition to the Best Estimate. It should be calculated using the Cost-of-Capital (CoC) methodology, based on the projection of the future regulatory capital requirements until the run-off of the liability portfolio, multiplied by the Cost-of-Capital rate (6%) and discounted using the risk-free interest rate term structure published by EIOPA.

The SCR is another important element in the quantitative requirements of Solvency II and represents the level of own funds that an insurer should hold, in order to absorb unexpected losses. It measures the capital needed to limit the probability of ruin to 0.5%, i.e. ruin would occur once every 200 years.

The SCR is calculated using the Value-at-Risk technique, for the one-year time horizon, either in accordance with the standard formula, or using an internal model, which will be further explained in Chapter 4. This amount aims to reflect the risk profile of the undertaking, taking into account all quantifiable risks to which the insurer is actually exposed to, as well as considering the impact of any risk mitigation instruments in place and diversification effects between risks.

The calibration based on a one-year time horizon does not mean ignoring the risks and cash flows occurring after this period of one year. It is assumed that the shock occurs within the next year, but the effects of it may propagate beyond this period and this is something that the SCR should capture as well.

Since the parameters of the standard formula are calibrated based on European data as a whole and given the diversity of markets and products, such calibration might not reflect the risk profile of a specific undertaking and consequently the level of own funds it needs given the risks' exposure. In case the insurer considers that such calibration is clearly inappropriate, it may request the supervisor's approval to use undertaking specific parameters (USP) in order to estimate those parameters using their own data. However such estimation is limited, once it must be done using standard methods defined by EIOPA.

3. STOCHASTIC METHODS FOR CLAIMS PROVISION

Claims reserving is a predictive process, which tries to estimate future claims based on past data. Currently, there are several methods to calculate claims provision and within them the stochastic methods stand out, since there seems to be clear advantages in their use by allowing measuring the volatility associated with claims reserving and therefore obtaining confidence intervals for the estimates.
In this chapter three different stochastic models for estimating claims provision are considered: Mack's model, Bühlmann-Straub's model and the Over-Dispersed Poisson Generalized Linear Model combined with the Bootstrap technique.

3.1. Mack's Model

There is a broad literature on the Mack's model therefore its presentation in this report is quite brief.
Thomas Mack developed in 1993 a distribution-free model whose results are identical to those obtained by the deterministic Chain Ladder method (CL), with the advantage of allowing the estimation of the mean squared error of Chain Ladder predictions. In this chapter, we present the assumptions underlying this model and afterwards explain how we can check if this model is appropriate towards a given dataset.

3.1.1. Model Assumptions

Let $A_{i,j}$ represent the cumulative claims amount of accident year $i, i = 0, 1, \dots, I$, either paid or incurred up to development year $j, j = 0, 1, \dots, I+1$, and f_j the respective development factor, $\hat{f}_j = \frac{\sum_{i=0}^{I-j-1} A_{i,j+1}}{\sum_{i=0}^{I-j-1} A_{i,j}}$, $0 \le j \le I-1$ and $\hat{f}_I = \frac{\hat{A}_{0,\infty}}{A_{0,I}}$ [1]. The amount $\hat{A}_{i,\infty}$ represents the total expected cumulative paid loss for each accident year i[2].
The assumptions of Mack's model are:

- $E(A_{i,j+1} | A_{i,0}, \dots, A_{i,j}) = A_{i,j} \cdot f_j, \quad 0 \le i \le I, \ 0 \le j \le I$;
- The variables $\{A_{i,0}, \dots, A_{i,\infty}\}$ and $\{A_{k,0}, \dots, A_{k,\infty}\}$, for different accidents years, $i \neq k$, are independents;
- $V(A_{i,j+1} | A_{i,0}, \dots, A_{i,j}) = \sigma_j^2 \cdot A_{i,j}, \quad 0 \le i \le I, \ 0 \le j \le I.$

[1] When we do not have an estimate for the ultimate amount, alternative methodologies to calculate $\hat{f}_I$ can be used. Like the one proposed in Mack (1999).

[2] By means of convention, the $A_{i,I+1}$ and $A_{i,\infty}$ amounts are equivalent.

3.1.2. Testing Model Assumptions

The verification of the first assumption implies the existence of proportionality between development years and the absence of correlation between the individual development factors $\hat{f}_{i,j} = \frac{A_{i,j+1}}{A_{i,j}}$, $0 \leq i \leq I, 0 \leq j \leq I$. One way to test the proportionality between development factors is to draw a straight line with slope f_j crossing the origin. If one observes significant differences between ordered pairs $(A_{i,j}, A_{i,j+1})$ and that line, the assumption should be rejected and one must review the estimation of f_j.

To test the lack of correlation between the individual development factors, we may consider the Spearman test. In order to apply it we begin to sort for a fixed development year j, $0 \leq j \leq I-1$ in ascending order the individual development factors $\hat{f}_{i,j}$, $0 \leq i \leq I-j-1$ denoting the order number assigned to $\hat{f}_{i,j}$ by $r_{i,j}$, $1 \leq r_{i,j} \leq I-j$. The same procedure is then applied to the precedent development factors $\hat{f}_{i,j-1} = \frac{A_{i,j}}{A_{i,j-1}}$, where the last factor $\frac{A_{I-j,j}}{A_{I-j,j-1}}$ is ignored, and denoting by $s_{i,j}$, $1 \leq s_{i,j} \leq I-j$ the respective order number. Then, we obtain an estimate for the Spearman correlation coefficient, T_j, using the following formula:

$$T_j = 1 - 6 \cdot \sum_{i=0}^{I-j-1} \frac{\left(r_{i,j} - s_{i,j}\right)^2}{((I-j)^3 - I + j)} \ , \quad 1 \leq j \leq I-2$$

where $-1 \leq T_j \leq 1$.

If T_j is close to zero it indicates no correlation between the development factors $\hat{f}_{i,j}$ and $\hat{f}_{i,j-1}$.

In order to consider the run-off triangle as a whole, reflecting any correlations that may exist, it was calculated the weighted average of T_j's, where the weights are inversely proportional to $V(T_j)$ (by this way we get a minimum variance estimator), and set T as:

$$T = \frac{\sum_{j=1}^{I-2}(I-j-1) \cdot T_j}{\sum_{j=1}^{I-2}(I-j-1)}$$

where, in the absence of correlations, we get $E(T) = 0$ and $V(T) = \frac{1}{\frac{(I-1)\cdot(I-2)}{2}}$

Given that T is obtained by averaging uncorrelated T_j's and the distribution of T_j, with $I - j \geq 10$, reasonably approximates the normal distribution, we assume that T also approaches the normal distribution. So we reject the assumption of no correlation between the individual development factors if the estimate of T does not fall within the following 50% confidence interval:

$$-\frac{0{,}6745}{\sqrt{\frac{(I-1)*(I-2)}{2}}} \leq T \leq \frac{0{,}6745}{\sqrt{\frac{(I-1)*(I-2)}{2}}}$$

It is used a confidence level of 50% instead of the usual 95%, since the test is displayed as an approximation and moreover it is intended to detect correlations in a significant part of the triangle.

The second assumption relies on the independence between different accident years. To examine whether some dataset verify this assumption, the following model is proposed: we start splitting the individual development factors in two sets, the highest and the lowest in value, for each year of development $j,\ 0 \leq j \leq I-1$. We denote by L the group comprising the higher individual development factors for each year of development and by S the group comprising the lower ones. The number of elements in each set should be equal, so if the number of factors in a given development year is odd, we eliminate the median value. Denoting D_k by the k^{th} diagonal of the individual development factors, i.e.:
$D_0 = \{\hat{f}_{0,0}\}$; $D_1 = \{\hat{f}_{0,1}, \hat{f}_{1,0}\}$; … ; $D_{I-1} = \{\hat{f}_{0,I-1}, \hat{f}_{1,I-2}, \hat{f}_{2,I-3}, \dots, \hat{f}_{I-1,0}\}$
The next step is, for each D_k diagonal, counting the number of elements belonging to L and S and denoting them by L_k and S_k, respectively. If $Z_k = \min(L_k, S_k)$ is significantly smaller than $\frac{(L_k+S_k)}{2}$, then there will be a prevalence of high or low factors in k^{th} diagonal and the independence assumption must be rejected.
Under the independence assumption, the variables L_k and S_k must follow a Binomial distribution with parameters $n = L_k + S_k$ and $p = 0{,}5$. By approaching the Binomial distribution to the Normal one, we should reject the second Mack's assumption, with a confidence level of 95%, if Z does not fall within the scope of the following confidence interval:

$$[\,E(Z) - 1{,}96 \cdot \sqrt{V(Z)}; E(Z) + 1{,}96 \cdot \sqrt{V(Z)}]$$

where $Z = Z_1 + Z_2 + \cdots + Z_{I-1}$ [3]

Finally, to verify the last assumption $V(A_{i,j+1} | A_{i,0}, \dots, A_{i,j}) = \sigma_j^2 \cdot A_{i,j}, \quad 0 \leq i \leq I$, $0 \leq j \leq I$, we should draw the ordered pairs $(\frac{A_{i,j+1} - A_{i,j} \cdot \hat{f}_j}{\sqrt{A_{i,j}}}, A_{i,j})$, by fixing j and then observe if the set of points does not show any trend. If that happens, we may not reject the assumption.

3.1.3. Variability Measures

Denoting $\hat{R}_i = \hat{A}_{i,\infty} - \hat{A}_{i,I-i}$ as the estimate for the provision that we should hold today to cover the losses incurred in the year i and $\hat{R} = \sum_{i=0}^{I} \hat{R}_i$ the estimate for the total provision, Thomas Mack defines the following variability measures:

[3] $E[Z_k] = \frac{n}{2} - \binom{n-1}{m} * \frac{n}{2^n}$ and $Var(Z_k) = \frac{n*(n-1)}{4} - \binom{n-1}{m} * \frac{n*(n-1)}{2^n} + E[Z_k] - E^2[Z_k]$ where m is largest integer $\leq \frac{n-1}{2}$
As under the null-hypothesis the variables $Z_k's$ are uncorrelated, we have: $E(Z) = \sum_1^{I-1} E(Z_k)$ and $V(Z) = \sum_1^{I-1} V(Z_k)$

$$\widehat{mse}\left(\hat{A}_{i,j+1}\right) = \hat{A}_{i,j}^2 \cdot \left(\widehat{mse}\left(\frac{A_{i,j+1}}{A_{i,j}}\right) + \widehat{mse}\left(\hat{f}_j\right)\right) + \widehat{mse}\left(\hat{A}_{i,j}\right) \cdot \hat{f}_j^2, \quad 0 \leq i \leq I$$

This is a recursive formula which allows us to obtain, when $j = I$, an estimate for $mse\left(\hat{R}_i\right) = \widehat{mse}\left(\hat{A}_{i,\infty}\right)$. The starting point for this recursive process will be $mse\left(\hat{A}_{i,I-i}\right) = 0$, since the $A_{i,I-i}$ amount is known.

$$\widehat{mse}\left(\frac{A_{i,j+1}}{A_{i,j}}\right) = \frac{\hat{\sigma}_j^2}{\hat{A}_{i,j}}, \qquad 0 \leq i \leq I, 0 \leq j \leq I-1$$

$$\widehat{mse}\left(\hat{f}_k\right) = \frac{\hat{\sigma}_j^2}{\sum_{i=0}^{I-j-1} A_{i,j}}, \quad 0 \leq i \leq I, 0 \leq j \leq I-1$$

The estimator $\hat{\sigma}_j^2$ is defined as:

$$\hat{\sigma}_j^2 = \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} A_{i,j} \cdot \left(\frac{A_{i,j+1}}{A_{i,j}} - \hat{f}_j\right)^2, \quad 0 \leq j \leq I-2$$

For the case $j = I - 1$, the following approximation is used:

$$\hat{\sigma}_{I-1}^2 = \min\left(\frac{\hat{\sigma}_{I-2}^4}{\hat{\sigma}_{I-3}^2}, min(\hat{\sigma}_{I-3}^2, \hat{\sigma}_{I-2}^2)\right)$$

If $\hat{f}_I = 1$, we can assume $\hat{\sigma}_I^2 = 0$ since it is not expected that claims payments will be paid over I years of development. When this is not the case and there are indexes $j-1$ and j such that $\hat{f}_j \leq \hat{f}_I \leq \hat{f}_{j-1}$, we can consider that the following inequations are also verified:

$$\widehat{mse}(\hat{f}_j) \leq \widehat{mse}(\hat{f}_I) \leq \widehat{mse}(\hat{f}_{j-1})$$

$$\widehat{mse}\left(\frac{A_{i,j+1}}{A_{i,j}}\right) \leq \widehat{mse}\left(\frac{A_{i,\infty}}{A_{i,I}}\right) \leq \widehat{mse}\left(\frac{A_{i,j}}{A_{i,j-1}}\right)$$

Taking this into consideration, we defined estimators for $\widehat{mse}(\hat{f}_I)$ and $\widehat{mse}\left(\frac{A_{i,\infty}}{A_{i,I}}\right)$ as:

$$\widehat{mse}\left(\hat{f}_I\right) = \widehat{mse}\left(\hat{f}_{j-1}\right) + \left(\widehat{mse}\left(\hat{f}_j\right) - \widehat{mse}\left(\hat{f}_{j-1}\right)\right) \cdot \left(\frac{\hat{f}_I - \hat{f}_j}{\hat{f}_{j-1} - \hat{f}_j}\right), \ \hat{f}_j \leq \hat{f}_I \leq \hat{f}_{j-1}$$

$$\widehat{mse}\left(\frac{A_{i,\infty}}{A_{i,I}}\right) = \widehat{mse}\left(\frac{A_{i,j}}{A_{i,j-1}}\right) + \left(\widehat{mse}\left(\frac{A_{i,j+1}}{A_{i,j}}\right) - \widehat{mse}\left(\frac{A_{i,j}}{A_{i,j-1}}\right)\right) \cdot \left(\frac{\frac{A_{i,\infty}}{A_{i,I}} - \frac{A_{i,j}}{A_{i,j-1}}}{\frac{A_{i,j}}{A_{i,j-1}} - \frac{A_{i,j+1}}{A_{i,j}}}\right),$$

$$\widehat{mse}\left(\frac{A_{i,j+1}}{A_{i,j}}\right) \leq \widehat{mse}\left(\frac{A_{i,\infty}}{A_{i,I}}\right) \leq \widehat{mse}\left(\frac{A_{i,j}}{A_{i,j-1}}\right)$$

The mean square error for the total provision is obtained using the following recursive formula, starting with $j = 0$:

$$\widehat{mse}\left(\sum_{i=I-j}^{I} \hat{A}_{i,j+1}\right) = \sum_{i=I-j}^{I} \hat{A}_{i,j}^2 \cdot \widehat{mse}\left(\frac{A_{i,j+1}}{A_{i,j}}\right) + \left(\sum_{i=I-j}^{I} \hat{A}_{i,j}\right)^2 \cdot \widehat{mse}\left(\hat{f}_j\right) +$$

$$+\widehat{mse}\left(\sum_{i=I-j+1}^{I} \hat{A}_{i,j}\right) \cdot \hat{f}_j^2$$

$\widehat{mse}\left(\hat{R}\right)$ is given when $j = I$, i.e., $\widehat{mse}\left(\hat{R}\right) = \widehat{mse}(\sum_{i=0}^{I} \hat{A}_{i,\infty})$

3.2. Bühlmann-Straub's Model

In 1970, Bühlmann and Straub have introduced a reasonably realistic model of claims evolution. Formally this model can be described as follows:
Conditional on an unobserved risk parameter Θ_i, the incremental claim payments $C_{i,j}, 0 \le i \le I, 0 \le j \le I+1$ have mean $E\left(C_{i,j}\middle|\Theta_i\right) = p_i \pi_j b(\Theta_i)$
The parameter p_i is defined as a measure of risk exposure in the accident year i. This quantity can be calculated through different ways, as long as one measures consistently over time. The parameter π_j specifies the expected proportion of ultimate claim cost that will be paid in development year j. We should assume that $\sum_{j=0}^{\infty} \pi_j = 1$. This parameter can be estimated using empirical development factors and the respective process to calculate it is explained in Neuhaus (2014).
The function $b(\Theta_i)$ denotes the claim rate in accident year i.

3.2.1. Model Assumptions

- Conditional on Θ_i, the increments $C_{i,0}, C_{i,1}, \ldots$ are stochastically independent.
- The sets $\{\Theta_i, C_{i,0}, C_{i,1}, \ldots\}$ and $\{\Theta_j, C_{k,0}, C_{k,1}, \ldots\}$ for different accidents years, $i \neq k$, are independent.
- The unobserved risk parameter Θ_i is the outcome of a random variable.
- The risks parameters $\Theta_1, \ldots, \Theta_I$ are assumed to be stochastically independent and identically distributed (i.i.d.)[4].

The model denotes the mean and variance of $b(\Theta_i)$ by $E\big(b(\Theta_i)\big) = \beta$ and $V\big(b(\Theta_i)\big) = \lambda$. Henceforth the mean of the function $v(\Theta_i)$ is denoted by $E\big(v(\Theta_i)\big) = \varphi$.

The general predictor for the future incremental payments is defined as $\bar{C}_{i,j} = p_i \bar{b}_i \pi_j,\ \ i+j > I$ where $\bar{b}_i$ is an estimator of $b(\Theta_i)$ and it is restricted to be a linear combination of a Chain Ladder estimate and the *a priori* mean β:

$$\bar{b}_i = z_i \hat{b}_i + (1 - z_i)\beta$$

The Chain Ladder estimate is given by:

$$\hat{b}_i = \frac{A_{i,I-i}}{p_i \pi_{\le I-i}}$$

The inequality in this subscript $(\le I - i)$ signifies the following summation: $\sum_{j=0}^{I-i} \pi_j$.

[4] Despite this last assumption does not represent the true reality in motor insurance, it will be very useful to support the empirical estimates set out below. Once we remove the i.i.d. assumption, it becomes more difficult to estimate the model parameters.

3.2.2. Variability Measures

Denoting the mean squared error of the estimator $\bar{b}_i$ by $Q(z_i)$, it is easy to verify that for an arbitrary choice of z_i :

$$Q(z_i) = E(\bar{b}_i - b(\Theta_i))^2 = {z_i}^2 \frac{\varphi}{p_i \pi_{\leq I-i}} + (1 - z_i)^2 \lambda$$

By minimizing $Q(z_i)$, we obtain an optimal choice of z_i, which is:

$$\xi_i = \frac{p_i \pi_{\leq I-i} \lambda}{p_i \pi_{\leq I-i} \lambda + \varphi}$$

The mean squared error of the predictor $\bar{C}_{i,j} = p_i \bar{b}_i \pi_j$ is:

$$E(\bar{C}_{ij} - C_{i,j})^2 = (p_i \pi_j)^2 Q(z_i) + p_i \pi_j \, \varphi$$

The mean squared error of the estimated provision $\hat{R}_i = \sum_{j=I+1-i}^{I+1} \bar{C}_{i,j}$ and $\hat{R} = \sum_{i=0}^{I} \hat{R}_i$ are given by:

$$E(\hat{R}_i - R_i)^2 = (p_i(1 - \pi_{\leq I-i}))^2 Q(z_i) + p_i(1 - \pi_{\leq I-i}) \, \varphi$$

and

$$E(\hat{R} - R)^2 = \sum_{i=1}^{I} E(\hat{R}_i - R_i)^2$$

According to Dubey & Gisler (1981), the model parameters estimates can be defined as follows:

$$\hat{v}_i = \frac{1}{I - 1 - i} \sum_{j=0}^{I-i} p_i \pi_j \left(\frac{C_{i,j}}{p_i j} - \hat{b}_i \right)^2, \qquad i = 0, \dots, I - 1$$

$$\hat{\varphi} = \frac{2}{I(I+1)} \sum_{i=0}^{I-1} (I - i) \hat{v}_i$$

$$\hat{\beta} = \xi^{-1} \sum_{i=0}^{I} \xi_i \, \hat{b}_i, \qquad \xi = \sum_{i=0}^{I} \xi_i$$

The estimator of β is the pseudoestimator weighted by ξ_i where φ and λ are replaced by estimators.

$$q = \sum_{i=0}^{I} p_i \pi_{\leq I-i} = \sum_{i=0}^{I} q_i$$

$$\hat{b} = \frac{\sum_{i=0}^{I} q_j \hat{b}_i}{q}$$

$$c = \frac{1}{q^2} (q^2 - \sum_{i=0}^{I} q_i^2)$$

$$\hat{\lambda} = \max(0, \frac{1}{c} \, (\sum_{i=0}^{I} \frac{q_i}{q} \left(\hat{b}_i - \hat{b}\right)^2 - \frac{I}{q} \hat{\varphi}))$$

3.3. Over-dispersed Poisson Model combined with the Bootstrap technique

3.3.1. Over-dispersed Poisson and Claims Reserves

GLMs, formulated by Nelder and Weddernurn in 1972, are a flexible generalization of ordinary linear regression models which are used to quantify and evaluate the relation between a response variable and a set of explanatory variables.
At this point, the theory underlying the GLM will be used to obtain estimates for claims provision. According to Renshaw and Verrall (1998), the over-dispersed Poisson (ODP) model is a stochastic model whose prediction estimates are very similar to the ones estimated by the CL method, with the advantage of obtaining estimates for the mean squared error of the predictions. Henceforth the i.i.d. response variables are represented by the incremental payments $C_{i,j}$ $0 \leq i \leq I,\ 0 \leq j \leq I+1$. In order to reproduce the same amounts estimated by the CL model, we should consider a GLM which assume that $C_{i,j}$ follows a Poisson distribution, a linear predictor of the form $\eta_{i,j} = \mu + \alpha_i + \beta_j$ and a logarithm link function.
The parameter μ assumes the claims occurred in year 0 and are paid in this year. The parameters α_i and β_j denote the effects of the i^{th} accident year and j^{th} development year on the expected value of $C_{i,j}$, respectively. In order to avoid over-parameterization, we need to restrict $\alpha_0 = \beta_0 = 0$.
However, there is an issue related to the fact that the chosen link function (logarithmic) implies the positivity of $C_{i,j}$ values which may not be reasonable to assume when it comes to claim amounts estimation. This problem is solved if, instead of using the pre-fixed scale parameter[5], we estimate a value for it. In this case, the variables $C_{i,j}$ are no longer Poisson distributed and therefore the parameters have to be estimated by maximum quasi-likelihood. Given that, the variables $C_{i,j}$ are now Over-dispersed Poisson distributed. This distribution differs from the Poisson distribution by the fact of the variance is not equal to the mean, but instead proportional, i.e.: $E(C_{i,j}) = \mu_{i,j}$ and $V(C_{i,j}) = \emptyset \cdot v(\mu_{i,j}) = \emptyset \cdot \mu_{i,j}$.
Renshaw and Verrall define also that it is necessary to impose the constraint that the sum of incremental claims for each development year is greater than 0.
Finally, to analyze the suitability of the model, i.e., to check if the model produces reliable estimates for the claims provision, it is convenient to plot the residuals versus the explanatory variables and analyze if they do not present any systemic pattern. Therefore, the Pearson residuals were applied. As it is well known the Pearson residual is defined as

$$r_{i,j}^{p} = \frac{C_{i,j} - \hat{\mu}_{i,j}}{\sqrt{\hat{v}(\mu_{i,j})\hat{\emptyset}}} \quad \text{where} \quad \hat{\emptyset} = \frac{1}{n-p} \sum_{i=0}^{I} \sum_{j=0}^{I-i} (r_{i,j}^{p})^2$$

[5] In the Poisson distribution, the scale parameter $\emptyset$ should be equal to 1.

3.3.2. Variability measures - Bootstrap Simulations

Obtaining estimates for the predictions errors of the above model is a more difficult task, therefore we decided to use the Bootstrap methodology. The Bootstrap method can be described as a resampling technique which repeatedly resamples the "original" data and in the end allows to make inferences from the resamples. Having a model which appropriately fits the data, the bootstrap technique can be applied to estimate, in a consistent way, the variability measures. In this context, the ODP will be the model used to obtain the future claims estimates.
For the application of the bootstrap, we need to have a set of observations of i.i.d. variables. Given the dependency between some observations and the parameter estimates, this resampling technique cannot be done directly from the observations $C_{i,j}$. It will then be applied to the residuals obtained by the ODP method. Such residuals should be independent and identically distributed. For facility, the Pearson residuals were used.
According to Pinheiro, Silva and Centeno (2003) it is indifferent to resample the residuals or the residuals multiplied by a constant, as long as we take that fact into account in the generation of the pseudo data. Since $\emptyset$ is a constant, we can take advantage from that and ignore the scale parameter and use the following Pearson residual formula:

$$r_{i,j}^{p} = \frac{C_{i,j} - \hat{\mu}_{i,j}}{\sqrt{\hat{\mu}_{i,j}}}, \quad 0 \leq i \leq I, \ 0 \leq j \leq I+1$$

It is simple to verify that the linear structure defined in the ODP implies some null residuals since we have $C_{0,I} = \hat{\mu}_{0,I}$, $C_{I,0} = \hat{\mu}_{I,0}$ and $C_{0,\infty} = \hat{\mu}_{0,\infty}$. According to Pinheiro, Silva and Centeno (2003), such residuals should not be considered as observations of the underlying random variable and consequently should not be considered in the bootstrap procedure.
In the following, we illustrate the steps of the bootstrap procedure:
Iteration No b

Step 1
Resampling of the Pearson residuals using replacement, creating a new past triangle of residuals $r_{i,j}^{b}$ where the indices i and j belong to the set $\{(i,j): 0 \leq i \leq I, 0 \leq j \leq I-1\} \cup \{(0, I+1)\}$.

Step 2
For each cell in the past triangle, create a set of pseudo-incremental data by solving:

$$C_{i,j}^{b} = r_{i,j}^{b} \cdot \sqrt{\hat{\mu}_{i,j}} + \hat{\mu}_{i,j}$$

Step 3
Apply the ODP model to this data set, obtaining estimates for the amounts still to

be paid, $\hat{\mu}_{i,j}^{b}$, and, consequently, for the provision for each accident year $\hat{R}_i^b = \sum_{j=I+1-i}^{I+1} \hat{\mu}_{i,j}^{b}$

Return to the beginning of step 1 until the B repetitions are completed.

Let's denote $\hat{R} = \sum_{i=1}^{I} \hat{R}_i$ as the estimate for the total provision.
The standard error estimates of $\hat{R}_i$ and $\hat{R}$ are given by

$$\hat{\sigma}_{bs}^2(\hat{R}_i) = \sqrt{\frac{1}{B}\sum_{b=1}^{B}\left(\hat{R}_i^b - \hat{R}_i^{bs}\right)^2}, \quad 0 \leq i \leq I$$

and

$$\hat{\sigma}_{bs}^2(\hat{R}) = \sqrt{\frac{1}{B}\sum_{b=1}^{B}\left(\hat{R}^b - \hat{R}^{bs}\right)^2}$$

Where B is the number of bootstrap simulations, $\hat{R}_i^b$ is the bootstrap estimate of R_i in the b-th replication $(1 \leq b \leq B)$ and $\hat{R}_i^{bs}$ and $\hat{R}^{bs}$ are, respectively, the mean of the B observations $\hat{R}_i^b$ and $\hat{R}^b$.
England and Verrall (1999) suggest a bias correction for the bootstrap standard error estimate to allow the comparison between the bootstrap standard error and the variability measures obtained analytically by the ODP GLM.
The bootstrap mean square error of prediction will be given by:

$$\widehat{mse}_{bs}(\hat{R}_i) = \hat{\emptyset} \cdot \hat{R}_i + \frac{n}{n-p}\hat{\sigma}_{bs}^2(\hat{R}_i), \quad 0 \leq i \leq I$$

and

$$\widehat{mse}_{bs}(\hat{R}) = \hat{\emptyset} \cdot \hat{R} + \frac{n}{n-p}\hat{\sigma}_{bs}^2(\hat{R}),$$

where
n is the number of observations and p is the number of parameters in the linear structure;

$$\hat{\emptyset} = \frac{1}{n-p}\sum_{i=0}^{I}\sum_{j=0}^{I-i}(r_{i,j}^{p})^2$$

4. SOLVENCY CAPITAL REQUIREMENT

The calculation of the SCR, according to the standard formula, is divided into modules as follows:

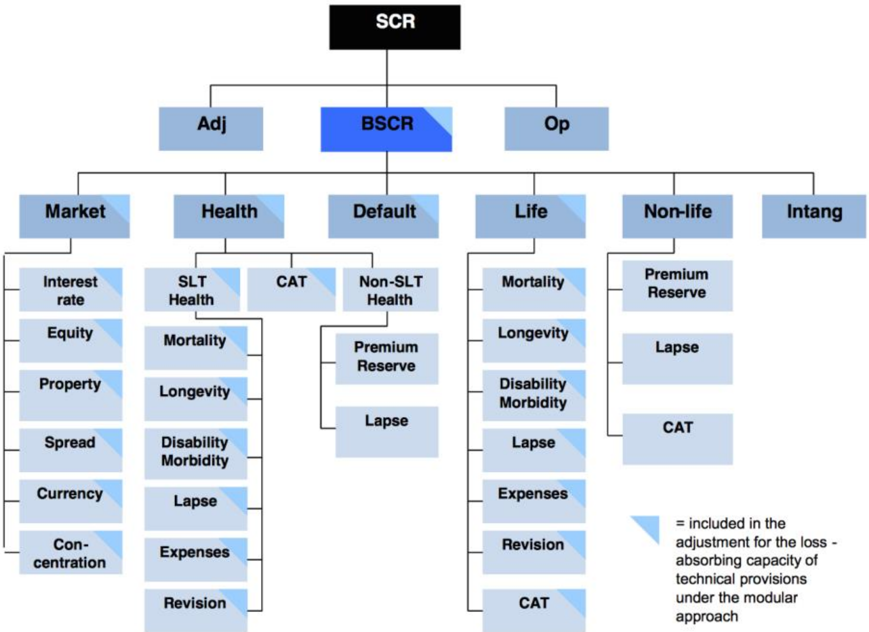


Figure 2: SCR modular structure

The SCR standard formula calculates the required regulatory capital using a modular approach, meaning that individual exposure to each risk category is assessed separately and then aggregated. For the aggregation of the individual risk modules to an overall SCR, it is assumed elliptical distributions and linear correlation techniques are applied[6].

The capital charges for each submodule are calibrated to reflect the VaR of 99.5% over a one-year time horizon. For each module, the respective submodules are then aggregated and after that the same process is repeated by aggregating the modules. Under the assumption of elliptical distributions, the result of the aggregations corresponds also to the VaR of 99.5% over a one-year time horizon.

Where the calibration of the standard formula is not considered as sufficiently appropriate, i.e. the risk profile of the undertaking deviates significantly from the assumptions underlying the standard formula calculation, the supervisor authority may, by means of a decision stating the reasons, require the undertaking to replace a set of parameters by USP or alternatively to use an internal model to calculate the SCR.

In order to better adjust the calculation of the SCR to their risk profile, undertakings may apply for approval to use one of the above mentioned as explained below:

- Subject to prior approval, insurance and reinsurance undertakings may re-

[6] The "modular" structure is further specified in Annex IV of the Solvency II Directive.

place a subset of the standard formula parameters by parameters specific to the undertaking. According to point (7) of article 104 of the Solvency II Directive, these parameters shall be calibrated on the basis of the internal data of the undertaking, or of data which is directly relevant for the operations of that undertaking.
The subset of standard parameters that may be replaced by USPs shall comprise the following sub-modules:

i. Non-Life premium and reserve risk sub-module;
ii. Life revision risk sub-module;
iii. Non-similar to life techniques (NSLT) health premium and reserve risk sub-module;
iv. Health revision risk sub-module.

For each of the parameters covered by the sub-modules, undertakings shall use the standardized methods set out in the Annex XVII of the Delegated Regulation. A credibility mechanism is applied in the scope of all of those methods. Depending on the number of years for which data are available, more or less weight is given to the undertaking specific parameter in relation to the standard parameter.
When granting supervisory approval, supervisory authorities shall verify the completeness, accuracy and appropriateness of the data used.

- Subject to approval by the supervisory authorities, undertakings may use full and partial internal models to calculate the SCR.
  Insurance and reinsurance undertakings may use partial internal models for the calculation of one or more risk modules or sub-modules of the SCR and the standard formula for the remaining ones.
  In addition, partial models may cover the whole business of the undertakings or only one or more major business units.
  The supervisor approval is subject to the assessment of six tests and standards that internal models should present and those are set out in articles 120 to 125 in the Delegated Regulation.
  Regarding the use of partial internal models, additional requirements are introduced to avoid cherry-picking by undertakings:
  (i) Proper justification for the limited scope of the model
  (ii) The resulting SCR reflects the risk profile of the undertaking more appropriately than the full use of the standard formula.
  Data used in the scope of internal models shall be accurate, complete, appropriate and it shall verify all the conditions established in Article 231 of the Delegated Regulation.

4.1. Reserve Risk

According to Article 105 of the Solvency II Directive, non-life underwriting risk module "shall reflect the risk arising from non-life insurance obligations, in relation to the perils covered and the processes used in the conduct of business". This risk module is composed by the following sub-modules: the non-life premium and reserve risk sub-module; the non-life catastrophe risk sub-module and the non-life lapse risk sub-module. However, in this report, due to its complexity and diversity of approaches that can be applied to the assessment of this risk in particular, only the reserve risk covered by the non-life premium and reserve risk sub-module is focused.

Reserve risk can be defined as the risk that the amount of current claims reserves is in reality insufficient to cover the run-off position, over a 12 month time horizon. With a little more precision, one-year reserve risk measures the variability underlying the estimates of ultimate loss that will be made within one year period.

The conceptual of reserve risk drivers are:

- Process risk: The process risk reflects the possibility of the absolute level of the ultimate unpaid loss be misestimated. Over the next 12 months the run-off of the reserves is uncertain once even if an appropriate estimation of the mean frequency and of the mean severity are applicable, the actual observed results will generally vary from the underlying means.
- Parameter risk: The parameter risk comes from the amount of information that will be gained over one year. The development factors estimated at the end of a period will differ from the development factors at the start of the period because of the additional information provided in this period. Consequently, this change will affect the ultimate unpaid loss estimate.
- Model Risk: This is the risk associated to the use of inaccurate models which causes a deviation in the expected value.

4.1.1. SCR Standard Formula for Reserve Risk

The SCR standard formula for premium and reserve risk was previously designed as follows:

$$SCR_{pr} = \rho(\sigma) \cdot V$$

σ denotes the combined standard deviation for non-life premium and reserve risk;
V denotes the total volume measure for non-life premium and reserve risk.
The function $\rho(\sigma)$ is specified as follows:

$$\rho(\sigma) = \frac{\exp(N_{0.995} \cdot \sqrt{\log(\sigma^2 + 1)})}{\sqrt{\sigma^2 + 1}} - 1$$

$N_{0.995}$ represents the 99.5% quantile of the standard normal distribution;
The function $\rho(\sigma)$, assuming a lognormal distribution of the underlying risk, is

consistent with the VaR 99.5% calibration.
Later on it was decided to approximate SCR_{pr} by $3 \cdot \sigma \cdot V$.
To calculate the combined standard deviation and the volume measure first we need to calculate them individually, for each LoB/segment, and then aggregate them using the formulae bellow.
According to article 116 of the Delegated Regulation, the volume measure of a particular segment *s* shall be equal to the following:

$$V = \sum_s V_s$$

$$V_s = V_{prem,s} + V_{res,s} \cdot (0{,}75 + 0{,}25 \cdot DIV_s)$$

where
$V_{prem,s}$ denotes the volume measure for premium risk of segment *s*;
$V_{res,s}$ denotes the volume measure for reserve risk of segment *s;*
DIV_s denotes the factor for geographical diversification of segment *s*, which can be ignored in the case of Portugal due to the reduced internationalization of the Portuguese insurance companies.
According to point 6 of article 116 of the Delegated Regulation, "the volume measure for reserve risk of a particular segment shall be equal to the best estimate of the provisions for claims outstanding for the segment, after deduction of the amounts recoverable from reinsurance contracts and special purpose vehicles, provided that the reinsurance contracts or special purpose vehicles comply with Articles 209, 210, 211 and 213. The volume measure shall not be a negative amount."
The standard deviation for non-life premium and reserve risk shall be equal to the following:

$$\sigma = \frac{1}{V} \cdot \sqrt{\sum_{s,t} CorrS_{s,t} \cdot \sigma_s \cdot V_s \cdot \sigma_t \cdot V_t}$$

where
V denotes the volume measure for non-life premium and reserve risk;
the sum covers all possible combinations (s, t) of the segments set out in Annex II of the Delegated Regulation;
$\mathrm{CorrS}_{s,t}$ denotes the correlation parameter for non-life premium and reserve risk for segment s and segment t, which is set out in Annex IV of the Delegated Regulation;
V_s and V_t denote the volume measures for premium and reserve risk of segments s and t, already defined above.
σ_s and σ_t denote standard deviations for non-life premium and reserve risk of segments s and t respectively and are defined as:

$$\sigma_s = \frac{\sqrt{\sigma_{prem,s}^2 \cdot V_{prem,s}^2 + \sigma_{prem,s} \cdot V_{prem,s} \cdot \sigma_{res,s} \cdot V_{res,s} + \sigma_{res,s}^2 \cdot V_{res,s}^2}}{V_{prem,s} + V_{res,s}}$$

where $\sigma_{prem,s}$ and $\sigma_{res,s}$ denote the standard deviations for non-life premium and reserve risk, respectively, of segment *s*.

Since we are only interested in studying the reserve risk of the Motor vehicle liability LoB, is simple to understand that from now on, the variables σ and V will represent, respectively, the standard deviation and the volume measure for reserve risk.

4.1.2. Standard Methods for Calculation of USP for Reserve Risk

In Annex XVII of the Delegated Regulation, two different models are presented for the calculation of the undertaking-specific standard deviation for reserve risk in the standard formula. This section provides a brief description of these two methods.

Method 1

The inputs under this method are:

- The sum of the best estimate provision at the end of the financial year t for claims that were outstanding at the beginning of the financial year and the payments made during the financial year for the same set of claims (this amount is denoted by y_t)
- Best estimate of the provision for outstanding claims at the beginning of financial year t (denoted by x_t)

This model is based on a set of assumptions being some of them the following:

- The amount denoted by y_t is linear proportional in the current best estimate of the provisions for the outstanding claims;
- The variance of amount denoted by y_t is quadratic in the current best estimate of the provisions for the outstanding claims;
- The amount denoted by y_t follows a lognormal distribution.

The undertaking-specific standard deviation for reserve risk, for each segment s, is estimated as:

$$\sigma_{(res,s,USP)} = c \cdot \hat{\sigma}\big(\hat{\delta}, \hat{\gamma}\big) \cdot \sqrt{\frac{T+1}{T-1}} + (1-c) \cdot \sigma_{(res,s)}$$

c denotes the credibility factor set out in section G of Annex XVII of the Delegated Regulation;

T denotes the latest financial year for which data are available;

$\sigma_{(res,s)}$ denotes the standard parameter that should be replaced by the undertaking-specific parameter;

$\hat{\sigma}\big(\hat{\delta}, \hat{\gamma}\big)$ denotes the standard deviation function set out as:

$$\hat{\sigma}\big(\hat{\delta}, \hat{\gamma}\big) = \exp\left(\hat{\gamma} + \frac{\frac{1}{2}T + \sum_{t=1}^{T} \pi_t(\hat{\delta}, \hat{\gamma}) \cdot \ln\left(\frac{y_t}{x_t}\right)}{\sum_{t=1}^{T} \pi_t(\hat{\delta}, \hat{\gamma})} \right)$$

The function π_t and its respective parameters $\hat{\delta}, \hat{\gamma}$ are defined in appendix E.

Method 2

The second method is based on the mean squared error of prediction (MSEP) of the one-year claims development result (CDR) giving by the Merz and Wüthrich (MW) method in 2008.

The CDR has been defined as an amount which allows us to compare two successive predictions of the total ultimate claim. MW defined the one-year claims development result as the difference between the estimated total ultimate claim at the beginning of the accounting year and the total estimated ultimate claim one year later. If the expected value of this difference is, at the beginning of the accounting year, zero (i.e. in average we neither expected losses nor gains), the MSE of this predicted value will be defined as:

$$msep_{\widehat{CDR}_i(I+1)|D_I}(0) = E\left[\left(\widehat{CDR}_i(I+1) - 0\right)^2 | D_I\right]$$

where

$\widehat{CDR}_i(I+1)$ represents the observable CDR for accident year i in accounting year $(I, i+1]$ and $D_I = \{C_{i,j}: i+j \leq I \; and \; i \leq I\}$ denote the claims data available at time I.

MW estimates this conditional MSEP at time I for the aggregated accident years as:

$$\widehat{msep}_{\sum_i \widehat{CDR}_i(I+1)|D_I}(0) =$$

$$= \sum_{i=1}^{I} \hat{A}_{i,I}^2 \cdot \frac{\frac{\hat{\sigma}_{I-i}^2}{\left(\hat{f}_{I-i}^{CL}\right)^2}}{A_{i,I-i}} + \sum_{i=1}^{I}\sum_{k=1}^{I}\left(\hat{A}_{i,I} \cdot \hat{A}_{k,I} \cdot \frac{\frac{\hat{\sigma}_{I-i}^2}{\left(\hat{f}_{I-i}^{CL}\right)^2}}{S_{I-i}} + \sum_{j=I-i+1}^{I-1} \frac{C_{I-j,j}}{S_j'} \cdot \frac{\frac{\hat{\sigma}_j^2}{\left(\hat{f}_j^{CL}\right)^2}}{S_j}\right)$$

where

$$S_j = \sum_{i=0}^{I-j-1} C_{i,j}$$

$$S_j' = \sum_{i=0}^{I-j} C_{i,j}$$

The estimates of the parameter σ_j^2 are defined according to the Mack's model.

According to method 2 the standard deviation for reserve risk, for each segment s, is defined as:

$$\sigma_{(res,s,USP)} = c \cdot \frac{\sqrt{\widehat{msep}_{\sum_i \widehat{CDR}_i(I+1)|D_I}(0)}}{BE} + (1-c) \cdot \sigma_{(res,s)}$$

Where BE is the current best estimate for outstanding claims estimated using the Chain Ladder method applied to paid claims development. The parameters c and $\sigma_{(res,s)}$ were already defined in method 1.

4.1.3. Internal Model for Reserve Risk

The most common stochastic methods used by insurance undertakings provide an ultimate view of the claims reserves volatility instead of a one-year view as required per Solvency II. The model defined in Wüthrich *et al.* (2008) is the first model to meet the one-year view in order to measure the volatility in claims reserves and it is the one applied in the scope of method 2 to estimate the USP, as previously mentioned.
Boumezoued, Angoua, Devineau and Boisseau (2011) provides an alternative methodology to the model defined in Wüthrich *et al.* (2008), measuring the uncertainty of claims reserves over a one-year time horizon, based on the bootstrap procedure. This method makes it possible to include a tail factor simulated in each bootstrap iteration, but, in this report we decided to use a simplified version and therefore we do not consider the existence of a tail.
When an internal model is applied, the goal is to evaluate the 99.5% quantile of the estimated CDR distribution. This allows the calculation of the capital requirement for the reserve risk in a stand-alone approach.
The one-year recursive bootstrap method, presented in the paper mentioned above, provides a full empirical distribution of the CDR.
In the following, we detail and illustrate the steps of this method. Step 1 is carried out only once while steps 2 to 7 are a bootstrap iteration.

Step 1
1.a. Calculation of the Chain Ladder individual development factors $(f_{i,j})_{0\leq i+j\leq I-1}$ and estimation of the parameters $(\hat{f}_j)_{0\leq j\leq I-1}$ and $(\hat{\sigma}_j)_{0\leq j\leq I-1}$.
1.b. Calculation of the current undiscounted reserve RE_I by

$$RE_I = \sum_{i=0}^{I} \hat{A}_{i,I} - A_{i,I-i}$$

1.c. Calculation of the adjusted residuals of the individual development factors by

$$r_{i,j} = \sqrt{\frac{I-j}{I-j-1}}\frac{\sqrt{A_{i,j}}(f_{i,j}-\hat{f}_j)}{\hat{\sigma}_j}, \qquad \forall\, i,j/: 0 \leq i+j \leq I-1$$

Iteration No b

Step 2
Resampling with replacement of the residuals in the upper triangle and obtaining pseudo-development factors:

$$f_{i,j}^b = r_{i,j}^b\sqrt{\frac{\hat{\sigma}_j^{\,2}}{A_{i,j}}} + \hat{f}_j, \qquad \forall\, i,j/: 0 \leq i+j \leq I-1$$

Step 3
Re-estimation of the Chain Ladder development factors by

$$f_j^b = \frac{\sum_{i=0}^{I-j-1} A_{i,j} \cdot f_{i,j}^b}{\sum_{i=0}^{I-j-1} A_{i,j}}, \qquad \forall\, j \in \{0, \dots, I-1\}$$

Step 4

Calculation of the expected cumulative payments in the sub-diagonal $A_{i,I+1-i}^b, \forall\, i \in \{1, \dots, I\}$ by simulating a normal distribution with mean $A_{i,I-i} \cdot f_{I-i}^b$ and variance $A_{i,I-i} \cdot \hat{\sigma}_{I-i}{}^2$. Simulation of one-year future payments by $P_{I+1}^b = \sum_{i=1}^{I} (A_{i,I+1-i}^b - A_{i,I-i}^b)$

Step 5

Calculation of new individual development factors $(f_{I-j,j}^b)_{0 \le j \le I-1}$ on the simulated sub-diagonal, and calculation of new Chain Ladder factors at the end of year $(I, I+1]$ by:

$$f_j^b = \frac{\sum_{i=0}^{I-j-1} A_{i,j} \cdot f_{i,j} + A_{I-j,j} \cdot f_{I-j,j}^b}{\sum_{i=0}^{I-j} A_{i,j}}, \quad \forall\, j \in \{0, \dots, I-1\}$$

Step 6

Calculation of the undiscounted reserve RE_{I+1} seen at time $I+1$ by

$$RE_{I+1}^b = \sum_{i=2}^{I} \hat{A}_{i,I}^b - A_{i,I+1-i}^b$$

with

$$\hat{A}_{i,I}^b = \left(\prod_{j=I-i+1}^{I-1} f_j^b \right) \cdot A_{i,I+1-i}^b, \qquad \forall\, i \in \{2, \dots, I\}$$

Step 7

Calculation of the CDR of iteration No b:

$$CDR^b = RE_{I+1}^b + P_{I+1}^b - RE_I$$

End of iteration No b.

Finally, when all the iterations are computed, the solvency capital requirement for the reserve risk is given by:

$$SCR_{reserve} = VaR_{99.5\%}(CDR)$$

5. RISK MARGIN

Article 77(3) of Solvency II Directive states that:

> *The risk margin shall be such as to ensure that the value of the technical provisions is equivalent to the amount an insurance/reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations in an arm's length transaction.*

The risk margin value should be calculated using the Cost-of-Capital (CoC) methodology, based on the projection of the future SCR until the run-off of the liability portfolio, multiplied by the Cost-of-Capital rate and then discounted using the risk-free interest rate term structure. The cost of capital methodology requires the risk margin to be calculated by determining the cost of providing an amount of eligible own funds equal to the SCR necessary to support the obligations over their lifetime.
All the assumptions regarding the risk margin calculation are presented in article 38 of the Delegated Regulation.
The overall risk margin should be calculated as follows:

$$RM = CoC \cdot \sum_{t \geq 0} \frac{SCR(t)}{(1 + r(t+1))^{t+1}}$$

CoC denotes the Cost-of-Capital rate. It will be the same for all (re)insurance undertakings and is currently 6%;
SCR(t) denotes the SCR, referred to in Article 38(2) of the Delegated Regulation, after t years;
$r(t + 1)$ denotes the basic risk-free interest rate for the maturity of t + 1 years.
Note that not all the risks are considered for the RM calculation since the future SCRs only capture the risks detailed in Article 38, point 1 (i) of the Delegated Regulation. As already referred, for the purpose of this reported we only considered the reserve risk.

5.1. Methodologies to Calculate the Risk Margin

Given the complexity in the projection of future SCRs, the precise calculation of the risk margin can be challenging. According to article 58 of the Delegated Regulation, undertakings may use simplified methods to calculate this technical provisions item. When deciding which simplified method should be selected to calculate the risk margin, the (re)insurer must ensure that the chosen method is adequate to the nature, scale and complexity of its risk profile. The complexity of the calculation of the risk margin should not go beyond that necessary to capture the risk profile of the undertaking.

According to guideline 61 of the set of EIOPA guidelines on the valuation of technical provisions, undertakings should use the following hierarchy of simplified methods, ranging from the most complex to the simplest, to calculate the Risk Margin:

1. Approximate the individual risks or sub-risks within some or all modules and sub-modules to be used for the calculation of future SCRs;
2. Approximate the whole SCR for each future year, inter alia by using the ratio of the best estimate at future year to the best estimate at the valuation date;
3. Estimate all future SCRs at once, inter alia by using the modified duration of the insurance liabilities as a proportionality factor;
4. Approximate the risk margin directly as a percentage of the best estimate.

Taking into consideration the simplifications mentioned above, in this section we propose two different methods to calculate the risk margin. These approaches are based on the cost-of-capital loading method and refer to the first two simplifications of the hierarchy above.

Method A

The second simplified method of the hierarchy is a commonly used simplification and therefore we decided to present it in this report. This simplification approximates the SCR for each future year t by multiplying the current SCR to the ratio defined as the best estimate at that future year over the best estimate at the valuation date, i.e.

$$SCR(t) = SCR(0) \cdot \frac{BE(t)}{BE(0)}$$

$SCR(0)$ represents the Solvency Capital Requirement at moment 0, used by the undertaking;
BE(0) represents the best estimate at the valuation date;
$BE(t)$ represents the best estimate technical provisions assessed at time $t \geq 0$.

Starting from the "last" best estimate, we have:

$$BE(I) = \frac{C_{I-1,\infty}}{(1 + f_{(I,I+1)})} + \frac{C_{I,\infty}}{(1 + f_{(I,I+1)}) \cdot (1 + f_{(I+1,I+2)})}$$

and

$$BE(I-1) = \frac{C(I-1) + BE(I) + C_{I-2,\infty}}{1 + f_{(I-1,I)}}$$

The remaining best estimates are obtained recursively from the above formula:

$$BE(t) = \frac{C(t-1) + BE(t+1) + C_{t-1,\infty}}{1 + f_{(t,t+1)}}$$

The variable $C(t+1)$ represents the sum of all paid amounts in the accounting year $t+1$.

The parameter $f_{(t,t+1)}$ represents the forward rate defined as:

$$f_{(t,t+1)} = \frac{(1+r(t+1))^{t+1}}{(1+r(t))^t} - 1$$

Finally, the risk margin would be:

$$RM = CoC \cdot \sum_{t\geq 0} \frac{SCR(t)}{(1+r(t+1))^{t+1}}$$

Method B

In the following method we are using the first simplification in the hierarchy, however we are only interested in approximating the SCR for the reserve risk sub-module.

Wüthrich *et al.* (2015) tries to complete the picture of uncertainty estimates in the CL claims reserving method by calculating the CDR behavior for the next accounting years. The developed prediction uncertainty formulas were used afterwards to estimate the RM item. In order to project the expected value of the claims development result mean square error for future years $t > I+1$, this paper starts by presenting the following formulas:

$$\widehat{\mathrm{E}}\left(msep^{MW}_{CDR_i(I+k+1)|D_{I+k}}(0)\middle|D_I\right) =$$
$$= (\hat{C}^{CL}_{i,J})^2 \frac{\sigma^2_{I-i+k}}{(\hat{f}^{CL}_{I-i+k})^2}\left(\frac{1}{\hat{C}^{CL}_{i,I-i+k}} + \prod_{m=1}^{k}(1-\alpha^{(I)}_{I-i+m})\frac{1}{\sum_{l=1}^{i-k-1} C_{l,I-i+k}}\right)$$
$$+ (\hat{C}^{CL}_{i,J})^2 \sum_{j=I-i+k+1}^{J-1} \frac{\sigma^2_j}{(\hat{f}^{CL}_j)^2}\left(\alpha^{(I)}_{j-k}\prod_{m=0}^{k-1}(1-\alpha^{(I)}_{j-m})\frac{1}{\sum_{l=1}^{I-j-1} C_{l,j}}\right),$$
$$k \geq 1$$

and

$$\widehat{\mathrm{E}}\left(msep^{MW}_{\sum_{i=k+1}^{I} CDR_i(I+k+1)|D_{I+k}}(0)\middle|D_I\right) =$$

$$\sum_{i=k+1}^{I} \widehat{\mathrm{E}}\left(msep^{MW}_{CDR_i(I+k+1)|D_{I+k}}(0)\middle|D_I\right) + 2\sum_{k+1\leq i\leq n\leq I} \hat{C}^{CL}_{i,I}\hat{C}^{CL}_{n,I}\frac{\sigma^2_{I-i+k}}{(\hat{f}^{CL}_{I-i+k})^2}\prod_{m=1}^{k}(1-\alpha^{(I)}_{I-i+m})\frac{1}{\sum_{l=1}^{i-k-1} C_{l,I-i+k}} + 2\sum_{k+1\leq i\leq n\leq I} \hat{C}^{CL}_{i,I}\hat{C}^{CL}_{n,I}\sum_{j=I-i+k+1}^{I-1}\frac{\sigma^2_{I-i+k}}{(\hat{f}^{CL}_{I-i+k})^2}\left(\alpha^{(I)}_{j-k}\prod_{m=0}^{k-1}(1-\alpha^{(I)}_{j-m}\frac{1}{\sum_{l=1}^{I-j-1} C_{l,j}}\right),\ k\geq 1$$

where

$\alpha^{(t)}_j = \frac{C_{t-j,j}}{\sum_{l=1}^{t-j} C_{l,j}} \in (0,1)$, $t = I,\dots,I+j$ and σ^2_j are the variance parameters in Mack's distribution-free CL method.

MW define these formulas as a result from a natural guess based on comparing the total run-off uncertainty formula proposed by Mack

$$msep^{Mack}_{C_{i,I}|D_I} = (\hat{C}^{CL}_{i,I})^2 \sum_{j=I-i}^{I-1} \frac{\sigma_j^2}{(\hat{f}^{CL}_j)^2}\left(\frac{1}{\hat{C}^{CL}_{i,j}} + \frac{1}{\sum_{l=1}^{I-j-1} C_{l,j}}\right)$$

and the one-year uncertainty formula proposed by Wüthrich *et al.* (2008), already mentioned in method 2 of section 4.1.12.. According to this paper, the difference between Mack's formula and the MW formula reflects the amount of prediction uncertainty which needs to be allocated to future years.

For the implementation of the above prediction uncertainty formulas, the R package ChainLadder, version 0.2.0 was used.

In section 7 of Wüthrich *et al.* (2015) three different methods to calculate the risk margin are presented. We decided to analyze and apply the second one with the standard deviation risk measure, which is given by:

$$kVar\left(\textstyle\sum_i CDR_{i,t+1}|D_t\right)^{1/2} = k\left(msep^{MW}_{\sum_i CDR_i(t+1|D_t)}(0)\right)^{1/2} \approx$$
$$k\widehat{\mathrm{E}}\left(msep^{MW}_{\sum_i CDR_i(t+1|D_t)}(0)\middle|D_I\right)^{\frac{1}{2}}, \mathrm{t} \geq I$$

where $k > 0$ is a given fixed loading constant.

This risk measure represents the required solvency capital that an undertaking needs to hold in order to absorb possible short-falls of the run-off in year $t+1$, i.e., the SCR(t+1). One could choose $k = 2.6$ for the standard deviation based risk measure, because it approximates to the 99.5% quantile of the standard Gaussian distribution, or $k = 3$ if a Lognormal distribution is assumed.

The risk margin for the standard deviation motivated risk is given by:

$$RM_t^{stdev} = \sum_{t\geq I} k \cdot CoC \cdot \widehat{\mathrm{E}}\left(msep^{MW}_{\sum_i CDR_i(t+1|D_t)}(0)\middle|D_I\right)^{\frac{1}{2}}$$

Since this formula does not consider any discount factor, we decided to introduce it in the form of:

$$RM_t^{stdev^*} = CoC \cdot \sum_{t\geq 0} \frac{k \cdot \widehat{\mathrm{E}}\left(msep^{MW}_{\sum_i CDR_i(t+1|D_t)}(0)\middle|D_I\right)^{\frac{1}{2}}}{(1+r(t+1))^{t+1}}$$

6. Results

The different proposed methodologies presented in the previous chapters were applied to a sample of data provided by ASF for Motor Vehicle Liability LoB. Such sample refers to a set of companies.The information on claims payments and reserves were used, both including the expenses incurred in servicing the insurance obligations, by accident and development year for claims occurred from 2005 to 2014. It is important to note that these historical reserves were valued according to the previous regime, Solvency I, and therefore these amounts may have embedded a higher prudency level. Despite these limitations, these data were used in the absence, for the time being, of historical Solvency II technical provisions.
The application of the theoretical models was performed using *Microsoft Excel* and the software *R*. The majority of the models were programmed in Visual Basic for Applications (VBA), only the method B was implemented using *R.* The full content of the VBA and R's coding can be available upon request.
For confidentiality reasons the run-off triangles will not be shown and therefore this chapter will only present the results related to the calculation of the best estimate and the (standalone) risk margin for claims provision and the reserve risk capital charge. In this chapter we also present confidence intervals for the undiscounted provisions values given by the models explored.

6.1. Results: Claims Provision

In order to calculate the best estimate for claims provision in the scope of Motor LoB, Vehicle Liability was split in Bodily Injury (BI) and Material Damages (MD) homogeneous risk groups (HRG). To calculate the provision amount using the BS model we need to define a measure of risk exposure. Since this LoB presents fairly homogeneous policies, we considered the number of policies as an adequate exposure measure.

Material Damages

As the BE values are a weighted average with no prudence attached, it makes more sense to calculate confidence intervals for the undiscounted provision. The table below presents the estimates of the standard errors and the confidence intervals with a significance level of 95% which were computed for each model.

	Undiscounted Provision	SE(Prov.)	%	95% CI Normal Dist.		95% CI Lognormal Dist.	
Mack	169.697.586	11.981.795	7,06%	146.213.700	193.181.473	147.424.593	194.366.618
Bühlmann Straub	171.626.607	7.276.729	4,24%	157.364.480	185.888.735	157.805.108	186.323.731
Bootstrap ODP	169.697.586	9.931.973	5,85%	150.231.276	189.163.897	151.062.616	189.980.580

Table 1: MD – Standard Errors and Confidence Intervals

The standard errors are relatively similar in the three methods, being the BS model the one with the smallest error and therefore a confidence interval with the smallest magnitude. The Mack model produced the highest results.

Since the reserve amounts given by Mack and OPD model are equal, we decided to join the results.
The BE results derived from the methods explained in chapter 3 are presented below:

	Mack and GLM-ODP	Bühlmann Straub
Undiscounted Provision (1)	169.697.586	171.626.607
BE(0) (2)	169.310.245	171.235.357
Discount Effect (3)=(2)-(1)	-387.341	-391.251
Discount Effect/Undiscounted Provision (4)=(3)/(1)	-0,2283%	-0,2280%

Table 2: MD - Best Estimates for Claims Provisions

In each of the three methods there is a low impact of discounting which is explained by the low level of discount rates observed at the reference date. For this purpose, the risk-free interest curves (annual zero-coupon spot rates) provided by EIOPA with reference date 31^{st} December 2014 was used. For our sample the results produced by the three models are quite close.

In order to analyze the sensitivity of the introduction of the discount effect, we calculate the impact on BE of applying the upward and downward shocks of the risk free rates (RFR) defined in articles 166 and 167 of the Delegated Regulation.

	Mack and GLM-ODP	Bühlmann Straub
BE(0) - RFR shock upwards (1)	166.657.408	168.554.715
Shock upwards effect (2)=(1)-Table I (2)	-2.652.838	-2.680.642
((Shock upwards effect/BE(0)) % (3)=(2)/Table I (2)	-1,567%	-1,565%
BE(0) - RFR shock downwards (4)	169.508.504	171.435.629
Shock downwards effect (5)=(4)-Table I (2)	198.259	200.272
(Shock downwards effect/BE(0)) % (6)=(5)/Table I (2)	0,1171%	0,1170%

Table 3: MD - Best Estimates with RFR Shocks

As expected, the three methods produce very similar results. The upward shock has a bigger impact on BE, since the difference between these rates and the basic RFR are much higher than the difference between the former and the rates with the downward shock.

For a more accurate estimation of the BE for claims provisions, we decided to apply an inflation rate adjustment. Knowing that the inflation rate has remained stable and close to 0 for the last years of our sample, we estimated the future inflation rates as 0.5% (inflation rate in 2015) and kept it constant.

	Mack and GLM-ODP	Bühlmann Straub
BE(0) without inflation adjustment (1)	169.310.245	171.235.357
BE(0) with infl. adj. (predicted future infl.=0.5%) (2)	165.690.618	169.938.358
Inflation effect (3)=(2)/(1)	-3.619.627	-1.296.999
Inflation effect/ BE(0) without adjustment (4)=(3)/(1)	-2,138%	-0,757%

Table 4: MD - Best Estimates with Inflation Adjustment

For the calculation of the BE without inflation adjustment it was implicitly assumed that future inflation rates were equivalent to the historical inflation rates which are higher than the current one.
The BS model produced the amount of the BE with the lowest impact resulting from the inflation adjustment.

Bodily Injury

The same calculations and analysis will now be performed considering the BI data sample.
Since the BI component has a longer/heavy tail, we decided to include a tail factor. Since, in one hand, the reserve values seemed quite high in comparison to payment values and on the other hand the tail development factor estimated through the presented claims provision methods is relatively low, we decided to calculate such parameter as:

$$\frac{A_{0,9} + Reserves}{A_{0,9}} \approx 1{,}08$$

Analyzing the development patterns in our sample, we decided to reduce the estimated tail factor towards 1.05. The cumulative paid loss $\hat{A}_{0,\infty}$ will be estimated as a CL projection, i.e., $\hat{A}_{0,\infty} = 1{,}05 \cdot A_{0,I}$.

Regarding the standard errors and confidence intervals, the same analysis is repeated here.

	Undiscounted Provision	SE(Prov.)	%	95% CI Normal Dist.		95% CI Lognormal Dist.	
Mack	625.496.795	27.878.922	4,46%	570.855.112	680.138.479	572.630.121	681.889.642
Bühlmann Straub	636.710.055	22.652.895	3,56%	592.311.196	681.108.914	593.461.449	682.246.825
Bootstrap ODP	625.496.795	34.378.053	5,50%	558.117.050	692.876.541	560.818.524	695.533.285

Table 5: BI - Standard Errors and Confidence Intervals[7]

Also here the standard errors of the three methods are relatively similar. The Bootstrap simulations produce the higher standard error and consequently the larger confidence interval whilst the BS model produced the smallest.
The BE results are presented in the following table. For discounting purposes it was assumed a duration of 2 years.

[7] For each application, the number of simulations used in the bootstrap method was 10.000.

	Mack and GLM-ODP	Bühlmann Straub
Undiscounted Provision (1)	625.496.795	636.710.055
BE(0) (2)	619.107.882	630.177.856
Discount Effect (3)=(2)-(1)	-6.388.913	-6.532.199
Discount Effect/Undiscounted Provision (4)=(3)/(1)	-1,021%	-1,026%

Table 6: BI - Best Estimates for Claims Provisions

As we can notice, the impact of discounting is lower in the Mack and ODP models than that of the BS model, however the difference is not significant.

The next table shows the impact on BE of applying the upward and downward shocks of the risk free rates.

	Mack and GLM-ODP	Bühlmann Straub
BE(0) - RFR shock upwards (1)	598.930.056	609.615.054
Shock upwards effect (2)=(1)-Table I (2)	-20.177.826	-20.562.802
((Shock upwards effect/BE(0)) % (3)=(2)/Table I (2)	-3,259%	-3,263%
BE(0) - RFR shock downwards (4)	621.645.375	632.769.119
Shock downwards effect (5)=(4)-Table I (2)	2.537.493	2.591.263
(Shock downwards effect/BE(0)) % (6)=(5)/Table I (2)	0,410%	0,411%

Table 7: BI - Best Estimates with RFR Shocks

From the above table we can see a higher impact caused by the increase of interest rates. This fact was already explained in the scope of the analysis of the results related to MD. Here as well, the results are very similar in the three methods.

In the following table it is presented the impact resulting from the inflation adjustment.

	Mack and GLM-ODP	Bühlmann Straub
BE(0) without inflation adjustment (1)	619.107.882	630.177.856
BE(0) with infl. adj. (predicted future infl.=0.5%) (2)	595.311.956	619.495.315
Inflation effect (3)=(2)/(1)	-23.795.926	-10.682.541
Inflation effect/ BE(0) without adjustment (4)=(3)/(1)	-3,844%	-1,695%

Table 8: BI - Best Estimates with Inflation Adjustment

The BE calculated by the BS model produced the lowest impact resulting from the inflation adjustment.

6.2. Results: SCR and Risk Margin

In order to obtain and compare a sufficiently wide range of results for the SCR and RM amounts, we applied method A using the different approaches presented in chapter 4 to calculate the current SCR. Method B is also applied to calculate the SCR and RM. We begin by presenting the results split in MD and BI.

Material Damages

Method A

In the scope of method A $SCR(0)$ can be calculated using different approaches. These results are presented in the first line of the following table calculated according to the standard formula either using the standard deviation or applying the USP for reserve risk.

$$\boldsymbol{SCR(0) = 3 \cdot \sigma \cdot BE(0)}$$

t	SCR(t) $\sigma = 0,09$	SCR(t) $\sigma = \sigma_{(USP,method1)} = 0,111$	SCR(t) $\sigma = \sigma_{(USP,method2)} = 0,074$
0	45.713.766	56.408.106	37.749.333
1	11.909.602	14.695.751	9.834.664
2	6.601.845	8.146.290	5.451.646
3	3.929.801	4.849.144	3.245.136
4	2.366.523	2.920.150	1.954.218
5	1.383.444	1.707.089	1.142.415
6	769.594	949.634	635.513
7	361.120	445.600	298.204
8	98.655	121.735	81.467
Risk Margin	4.376.025	5.399.758	3.613.617

Table 9: MD - Method A - $SCR(0)$ calculation using the standard formula

Since the reserve amounts used in method 1 carries extra information and are based in the previous regime, a relatively high value for the USP resulting from this method is expected. Such result is then reflected in the $SCR(0)$ value, which is the highest between the three approaches. The risk parameter value given by method 2 is smaller than the standard risk parameter. This difference can be justified by the fact that the standard parameter was calibrated on an European basis which may not be the most appropriate to reflect the underlying risk profile of our sample. The $BE(t)$ values used in model A were calculated according to the Thomas Mack model and because of that the extrapolated $SCR(t)$ values follows the same pattern as the $SCR(0)$.

The $SCR(0)$ value in the next table was calculated using the method presented in section 4.1.3.

$$\boldsymbol{SCR(0) = VaR_{99.5\%}(CDR_{\ Bootstrap})}$$

t	SCR(t)
0	42.789.972
1	11.147.879
2	6.179.600
3	3.678.456
4	2.215.163
5	1.294.961
6	720.372
7	338.023
8	92.346
Risk Margin	4.096.140

Table 10: MD - Method A - $SCR(0)$ calculation using the bootstrap technique

Both $SCR(0)$ and RM values are close to the values in second and last column of table 9.

Method B

t	SCR(t) Lognormal Dist.	SCR(t) Normal Dist.
0	35.030.978	30.360.181
1	7.176.285	6.219.447
2	2.563.481	2.221.684
3	1.883.208	1.632.114
4	1.452.889	1.259.170
5	811.043	702.904
6	615.863	533.748
7	332.347	288.034
8	160.800	139.360
Risk Margin	2.993.719	2.594.556

Table 11: MD - Method B

The amounts for SCRs and RM given by this method are small comparatively with the above results. Since method B uses the first approach in the hierarchy of simplified methods, we could assume that the results coming from this method are more accurate than the above ones.

Bodily Injury

Next, the same calculations and analysis will be repeated considering the bodily injury data sample. As an example, we decided to include an estimated tail factor only in method A when the $SCR(0)$ is calculated according to the standard formula.

Method A

$$\boldsymbol{SCR(0) = 3 \cdot \sigma \cdot BE(0)}$$

t	SCR(t) $\sigma = 0,09$	SCR(t) $\sigma = \sigma_{(USP,method1)} = 0,07$	SCR(t) $\sigma = \sigma_{(USP,method2)} = 0,048$
0	167.159.128	129.549.851	88.321.432
1	137.887.296	106.863.914	72.855.150
2	100.632.669	77.991.238	53.171.020
3	72.828.927	56.443.084	38.480.430
4	51.892.343	40.217.040	27.418.222
5	35.429.520	27.458.201	18.719.803
6	22.682.848	17.579.414	11.984.877
7	13.319.785	10.322.955	7.037.740
8	5.970.885	4.627.491	3.154.821
9	2.497.943	1.935.928	1.319.831
Risk Margin	36.358.792	28.178.396	19.210.800

Table 12: BI - Method A - $SCR(0)$ calculation using the standard formula

Once again, method 2 produced the lowest risk parameter and therefore the last column presents the smallest values. Contrary to what happen in material damages data, method 1 produced an USP higher than the standard risk parameter which is then reflected in the $SCR(t)$ and RM values.

$$SCR(0) = VaR_{99.5\%}(CDR_{Bootstrap})$$

t	SCR(t)
0	90.743.229
1	74.852.858
2	54.628.984
3	39.535.573
4	28.170.036
5	19.233.105
6	12.313.505
7	7.230.717
8	3.241.327
Risk Margin	19.661.859

Table 13: BI - Method A – $SCR(0)$ calculation using the bootstrap technique

The $SCR(0)$ produced based on the bootstrap technique is quite close to the one given by the standard formula using the method 2 risk parameter. This leads us to believe that there would be a benefit to replace the reserve risk parameter by USP or to use the methodology presented in section 4.1.3. to calculate the $SCR(0)$. The RM value is also very similar to the last column of table 12, as expected.

Method B

t	SCR(t) Lognormal Dist.	SCR(t) Normal Dist.
0	57.033.153	49.428.733
1	40.117.206	34.768.245
2	22.063.518	19.121.716
3	16.243.317	14.077.541
4	11.485.443	9.954.051
5	8.315.163	7.206.475
6	6.809.376	5.901.459
7	5.271.675	4.568.785
8	3.600.510	3.120.442
Risk Margin	10.182.647	8.824.961

Table 14: BI – Method B

Comparing with method A, the results are significantly smaller and the absence of a tail factor is immaterial in explaining such deviation. It seems to lead to the conclusion that the previous results may be overestimating the risk associated once in theory this method should produce more accurate results as long as the data is sufficiently reliable and representative of the underlying risk.

If instead of a pure HRG analysis, there is the need to produce results for the whole LoB but at the same time making use of the insights gained through this more detailed and risk oriented assessment (per HRG), the previous results need to be aggregated. However, since the BI triangles have a longer tail and the payments volume has a smaller weight in the first development years comparing to MD, according to Ajne (1994) it would not be appropriate to simply add up the corresponding chain-ladder projections of the individual data sets. For this reason, in order to obtain the overall standard deviation, the standard deviations for each HGR was performed separately and the correlation of these two components was estimated.

In order to do so, the Pearson correlation was computed between the estimated CDR distributions given by the bootstrap method presented in section 4.1.3.. The correlation obtained was 0.268.
For this purpose, the results of Method A are presented below.
The $BE(t), t \geq 0$ used in this method is simply the sum of the best estimates for the MD and BI.

$$\boldsymbol{SCR(0) = 3 \cdot \sigma \cdot BE(0)}$$

t	SCR(t) $\sigma = 0,09$	SCR(t) $\sigma = \sigma_{(USP,method1)} = 0,146$	SCR(t) $\sigma = \sigma_{(USP,method2)} = 0,101$
0	212.872.894	345.944.780	237.741.234
1	149.796.898	243.438.485	167.296.544
2	107.234.514	174.269.348	119.761.916
3	76.758.728	124.742.426	85.725.873
4	54.258.865	88.177.367	60.597.521
5	36.812.964	59.825.619	41.113.545
6	23.452.442	38.113.119	26.192.214
7	13.680.905	22.233.162	15.279.142
8	6.069.541	9.863.754	6.778.599
9	2.497.943	4.059.466	2.789.758
Risk Margin	40.734.817	66.199.115	40.581.828

Table 15: MD and BI - Method A - $SCR(0)$ calculation using the standard formula

The method 2 factor is very close to the standard parameter and significantly better/lower than the one produced by method 1. In light of these outcomes it would be suitable to use the standard deviation value.

For the following approach, we calculated separately the CDR distributions and apply the 99.5% percentile to the sum of the bootstrap simulations.

$$\boldsymbol{SCR(0) = VaR_{99.5\%}(CDR_{\,Bootstrap})}$$

t	SCR(t)
0	109.915.752
1	77.346.807
2	55.369.954
3	39.633.949
4	28.016.268
5	19.008.172
6	12.109.540
7	7.064.060
8	3.133.974
Risk Margin	20.961.191

Table 16: MD and BI - Method A – $SCR(0)$ calculation using the bootstrap technique

This approach produced results that are sufficiently different from that of the standard formula. The internal model is applied for SCR calculation, however, the RM value will naturally be affected since its formula depends on the SCR values. Given that the above results are considerable low comparatively with the ones in table 15, this leads us to think that would it be appropriate to explore the idea of applying an internal model to this line of business.

7. CONCLUSION

The aim of this work was to implement some of the well-known claims provision methods considering the Solvency II valuation principles as well as the calculation of the reserve risk capital charge using either the standard formula with and without undertaking specific parameters (USP) or an internal model. At the end of this document it is also focused the RM item with two different approaches to calculate it. A specific line of business (LoB) - the Motor Vehicle Liability insurance was focused envisaging the presentation of a possible application of the theoretical methods analyzed. The LoB was split in Bodily Injury (BI) and Material Damages (MD) homogeneous risk groups (HRG).

The literature on claims reserving is very much diversified but due to time constraints, we decided to focus on the methods more commonly used and explored. For our sample the claims provision selected methods produced quite similar results. Those values were then appropriately discounted in order to have a best estimate value as required in the current regime. In each of the three models (Mack's model, Bühlmann-Straub model and the Over-Dispersed Poisson Generalized Linear Model), some sensitivity analysis were accomplished as the impact on BE of applying the upward and downward shocks of the RFR and an inflation rate adjustment. It was observed a low impact of discounting which is explained by the low level of discount rates observed at the reference date. Since the inflation rate has remained stable and close to 0 for the last years of our sample, we estimated the future inflation rates as 0.5% (inflation rate in 2015) and kept it constant. Here it was observed a considerable (negative) impact in the amount of the BE comparing to a calculation where it was implicitly assumed that future inflation rates were equivalent to the historical inflation rates which are higher than the current one.

In order to calculate the SCR for reserve risk besides the standard formula including the application of the standardized methods for the calculation of USP and the bootstrap technique which provided a full empirical distribution of the CDR were implemented. For the first years of implementation of Solvency II, the standard method 2 for the calculation of USP could be more appropriate than the method 1 since the latter depends on the historical provisions (Solvency I) rather than best estimates. In the scope of our sample, moving from the pure standard formula to an approach where USP was used and again to our internal model proposed methodology, means decreasing the required capital charge. This leads us to believe that there would be a benefit to use the bootstrap methodology to calculate the $SCR(0)$ whilst potentially benefitting from a more appropriate model to explain the underlying risk profile considering that it better reflects the data available and does not depend on a theoretical distribution assumption that has not been tested to our particular case but rather established for the whole European market. On the other hand, when the two HRGs are aggregated, the results obtained by the standard methods for the calculation of USP are higher than the result given by the standard formula. In particular the risk parameter given by method 1 is almost one basis point above the standard. Also here the bootstrap technique produced the lowest results.

Given the complexity and the uncertainty involved in projection future SCRs, the precise calculation of the risk margin can be challenging. In this work we present two approaches based on the cost-of-capital method which refer to the first and second simplifications of the hierarchy of simplified methods to calculate the RM set out in EIOPA guidelines on the valuation of technical provisions. The complexity of those methods decreases from the top to the bottom being why we decided to focused on the two more sophisticated approaches. The results obtained by the first simplification were always the lowest, which seems to lead to the conclusion that the results given by the second simplification may be overestimating the risk associated once in theory the first simplification should produce more accurate results.

Further developments for this work would be:

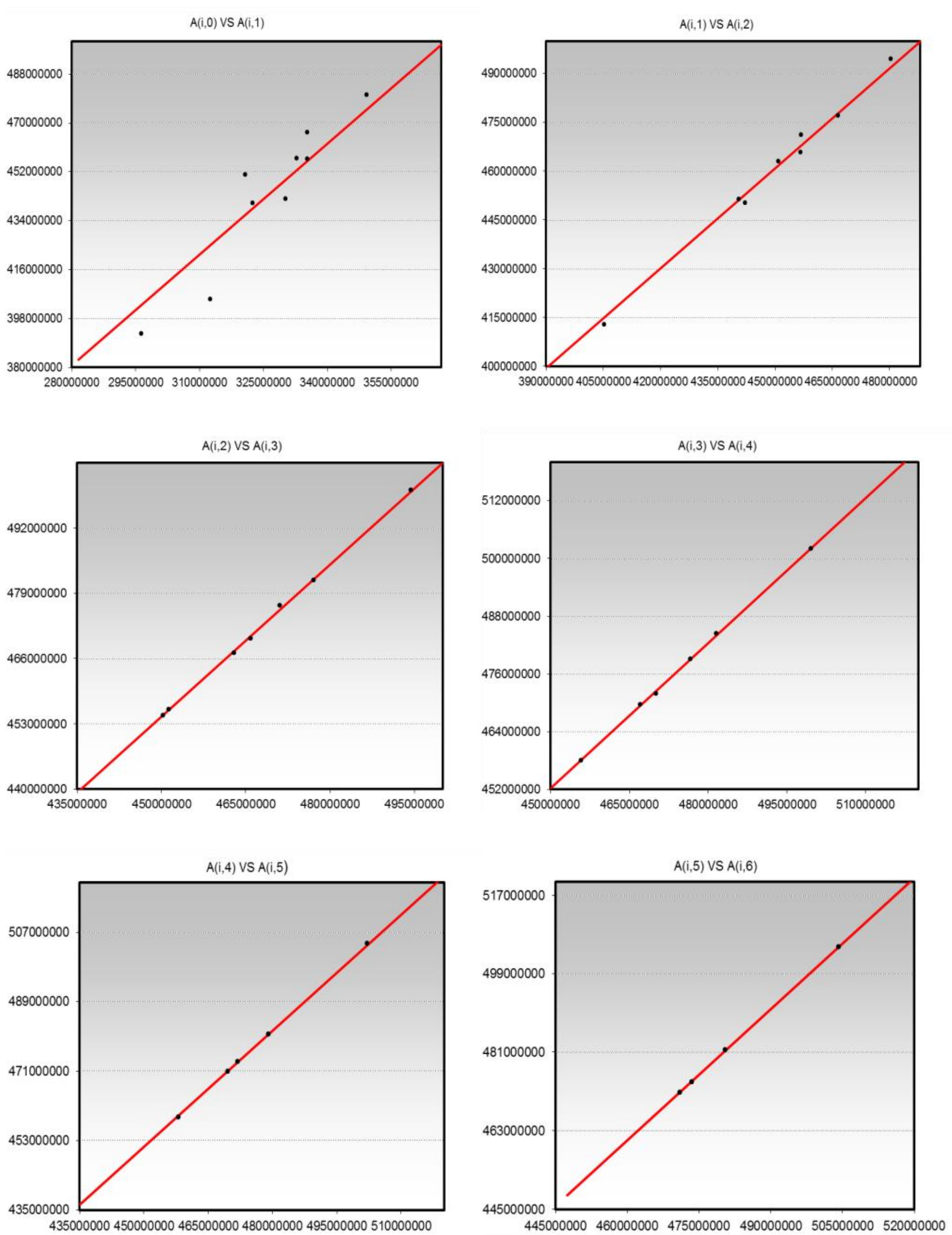
1. Explore other partial internal models to calculate reserve risk;
2. Study the premium provisions and the premium risk and combine the obtained results with that of for reserve risk.

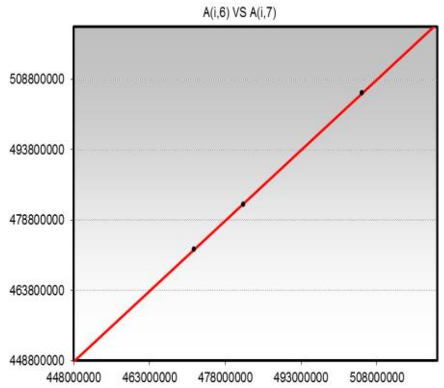
APPENDIX A – Mack's Model Validation of Assumptions

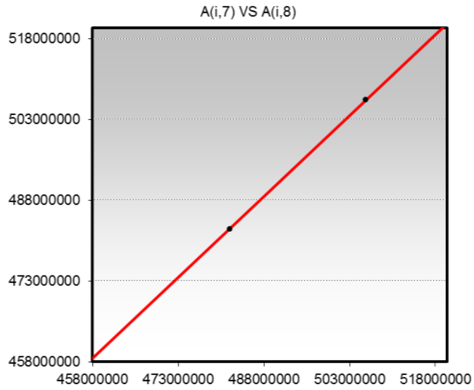
A.1. First Assumption

i. Proportionality between development years

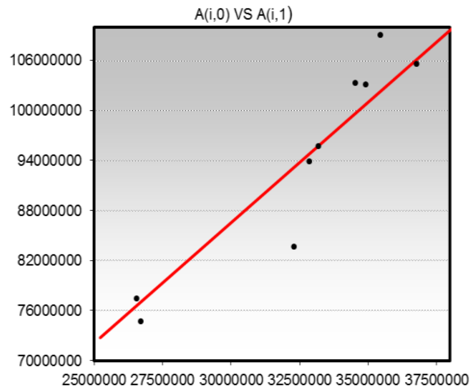
Material Damages

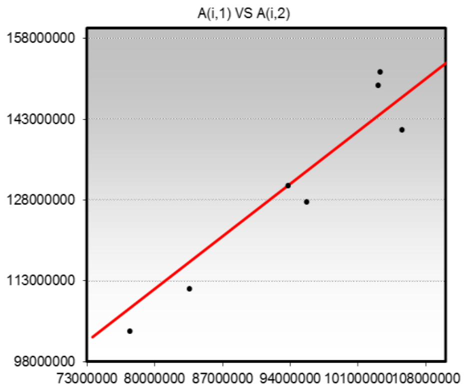


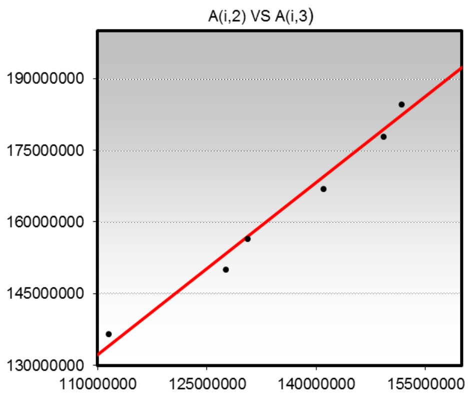


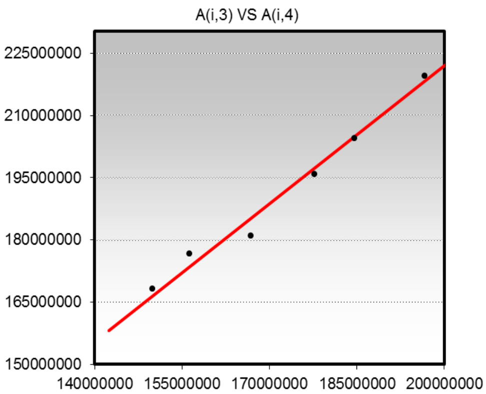


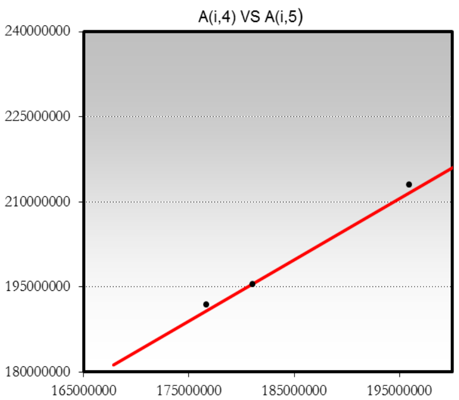
Bodily Injury

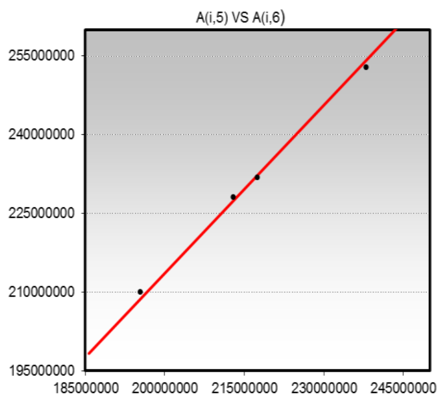


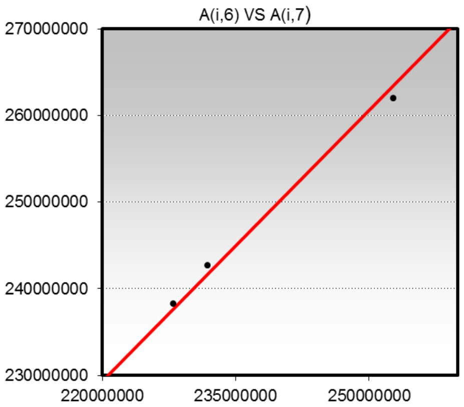


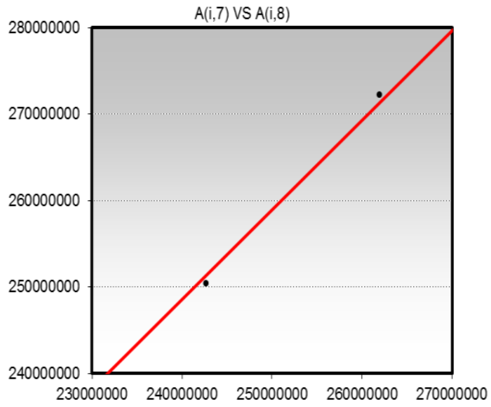












As we can see, for each development year j, the line with slope f_j , seems to fit quite well to the observed data. We can thus assume the existence of proportionality between adjacent development years is verified for these data sets.

ii. Absence of correlation between individual development factors – Spearman Test

Material Damages

Individual Development Factors

	0	1	2	3	4	5	6	7	8	9	∞
2005	1,3750	1,0293	1,0105	1,0050	1,0042	1,0017	1,0016	1,0016	1,0009	1,0000	
2006	1,3726	1,0311	1,0117	1,0053	1,0030	1,0021	1,0009	1,0014			
2007	1,4058	1,0267	1,0090	1,0054	1,0028	1,0018	1,0014				
2008	1,3621	1,0200	1,0089	1,0040	1,0034	1,0014					
2009	1,3661	1,0247	1,0099	1,0048	1,0021						
2010	1,3910	1,0228	1,0094	1,0059							
2011	1,3389	1,0184	1,0096								
2012	1,2967	1,0190									
2013	1,3242										
2014											

Ascending order of $r_{i,j}$ values

	0	1	2	3	4	5	6	7	8	9	∞
2005		7	6	3	5	2	3	2	1		
2006		8	7	4	3	4	1	1			
2007		6	2	5	2	3	2				
2008		3	1	1	4	1					
2009		5	5	2	1						
2010		4	3	6							
2011		1	4								
2012		2									
2013											
2014											

Ascending order of $s_{i,j}$ values

	0	1	2	3	4	5	6	7	8	9	∞
2005		6	6	5	3	4	1	2	1		
2006		5	7	6	4	2	3	1			
2007		8	5	2	5	1	2				
2008		3	2	1	1	3					
2009		4	4	4	2						
2010		7	3	3							
2011		2	1								
2012		1									
2013											
2014											

T_j estimates are:

j	1	2	3	4	5	6	7
T_j	2/3	2/3	1/7	- 1/5	- 3/5	-1	1

The estimate for T is given by:

$$T = 0{,}20731$$

The confidence interval for the estimate T is:

$$[-0{,}12750\ ;\ 12{,}747]$$

Since our estimate is inside of the range, we should not reject the assumption about no correlation between the development factors. However we still decided to apply the Mack's model to this data set given the main purpose of the Project.

Comments and suggestions are repeated here for BI.

Bodily Injury

Individual Development Factors

	0	1	2	3	4	5	6	7	8	9	∞
2005	3,0720	1,4700	1,2278	1,1159	1,0844	1,0620	1,0364	1,0393	1,0223	1,0500	
2006	2,9909	1,4696	1,2158	1,1083	1,0634	1,0661	1,0467	1,0318			
2007	2,9523	1,4483	1,1907	1,1019	1,0871	1,0705	1,0448				
2008	2,8700	1,3360	1,1824	1,0855	1,0795	1,0740					
2009	2,8558	1,3919	1,1968	1,1303	1,0865						
2010	2,8843	1,3338	1,1744	1,1216							
2011	2,5882	1,3337	1,2238								
2012	2,9172	1,3377									
2013	2,7955										
2014											

Ascending order $r_{i,j}$ of values

	0	1	2	3	4	5	6	7	8	9	∞
2005		8	7	4	3	1	1	2	1		
2006		7	5	3	1	2	3	1			
2007		6	3	2	5	3	2				
2008		3	2	1	2	4					
2009		5	4	6	4						
2010		2	1	5							
2011		1	6								
2012		4									
2013											
2014											

Ascending order of $s_{i,j}$ values

	0	1	2	3	4	5	6	7	8	9	∞
2005		8	7	6	4	3	1	1	1		
2006		7	6	5	3	1	2	2			
2007		6	5	3	2	4	3				
2008		3	3	2	1	2					
2009		2	4	4	5						
2010		4	2	1							
2011		1	1								
2012		5									
2013											
2014											

The T_j estimates are:

j	1	2	3	4	5	6	7
T_j	5/6	3/7	1/7	1/5	0	1/2	-1

The estimate for T is given by:

$$T = 0{,}354252$$

The confidence interval for the estimate T is:

$$[-0{,}127468519\ ;\ 0{,}127468519]$$

Since our estimate is outside of the range, we should reject the assumption about no correlation between the factors of development. Also here, we opted to trust our data and apply the Mack's model to the BI data set.

A.2. Second Assumption

Independence between different accident years

Material Damages

Classification of the individual development factors

	0	1	2	3	4	5	6	7	8	9
2005	L	L	L	S	L	S	*	L	*	
2006	L	L	L	L	*	L	S	S		
2007	L	L	S	L	S	L	L			
2008	S	S	S	S	L	S				
2009	*	L	L	S	S					
2010	L	S	S	L						
2011	S	S	*							
2012	S	S								
2013	S									
2014										

The notation "*" represents the median values which are ignored.

k	S_k	L_k	Z_k	n	m	$E(Z_k)$	$V(Z_k)$
0	0	1	0	1	0	0	0
1	0	2	0	2	0	0,50	0,25
2	0	3	0	3	1	1,125	0,61
3	2	2	2	4	1	1,83	0,72
4	2	2	2	4	1	1,83	0,72
5	2	3	2	5	2	0	3,75
6	4	2	2	6	2	2,88	0,90
7	5	3	3	8	3	3,78	1,95
8	5	2	2	7	3	3,23	8,86
Total			**13**			**15,18**	**17,77**

The confidence interval with a significance level of 95% for Z is:

$$[6{,}91772\ ;\ 23{,}44228]$$

Since the $Z = \sum_k Z_k = 13$ estimate is inside the interval, we accept the assumption of independence between different accident years.

Bodily Injury

Classification of the individual development factors

	0	1	2	3	4	5	6	7	8	9
2005	L	L	L	L	*	S	S	L	*	
2006	L	L	L	S	S	S	L	S		
2007	L	L	S	S	L	L	*			
2008	S	S	S	S	S	L				
2009	S	L	*	L	L					
2010	*	S	S	L						
2011	S	S	L							
2012	L	S								
2013	S									
2014										

k	S_k	L_k	Z_k	n	m	$E(Z_k)$	$V(Z_k)$
0	0	1	0	1	0	0	0
1	0	2	0	2	0	0,50	0,25
2	0	3	0	3	1	1,125	0,61
3	1	3	1	4	1	1,83	0,72
4	4	0	0	4	1	1,83	0,72
5	4	1	1	5	2	0	3,75
6	5	1	1	6	2	2,88	0,90
7	3	5	3	8	3	3,78	1,95
8	3	4	3	7	3	3,23	8,86
Total			**9**			**15,18**	**17,77**

The confidence interval with a significance level of 95% for Z is:

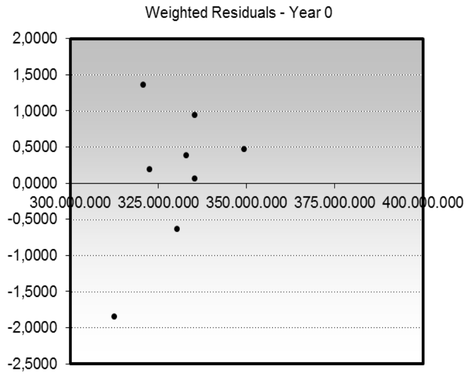
$$[6{,}917722348\ ;\ 23{,}44227765]$$

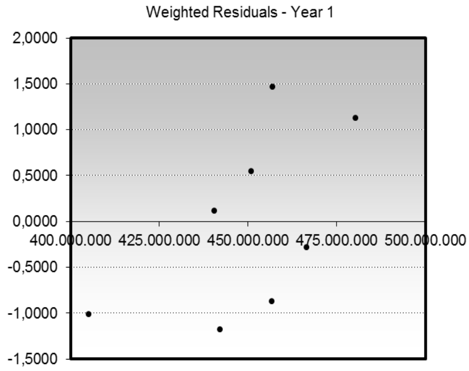
Since the $Z = \sum_k Z_k = 9$ estimate is inside the interval, we accept the assumption of independence between different accident years.

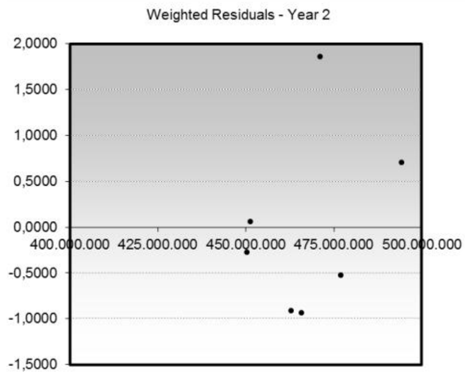
A.3. Third Assumption

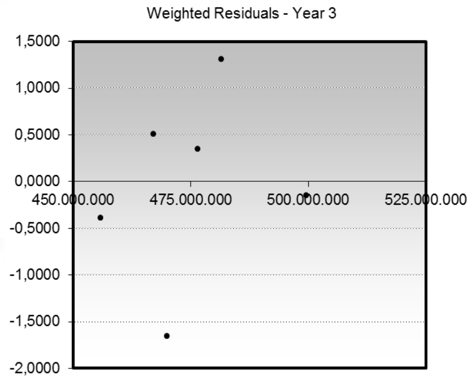
Weighted Residuals

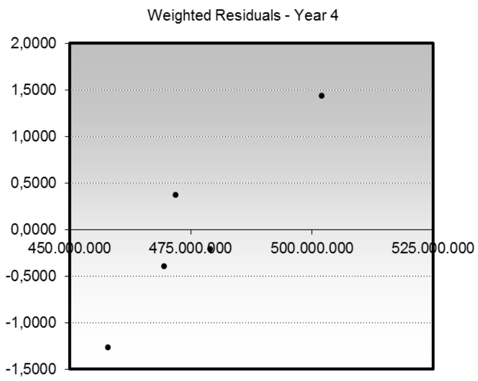
Material Damages

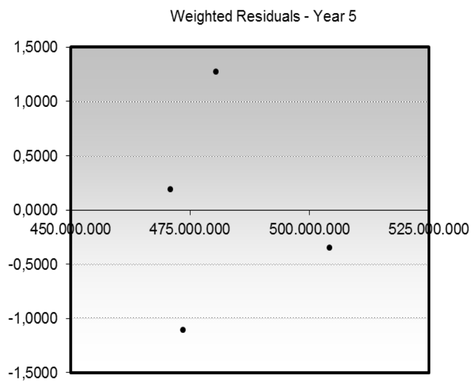


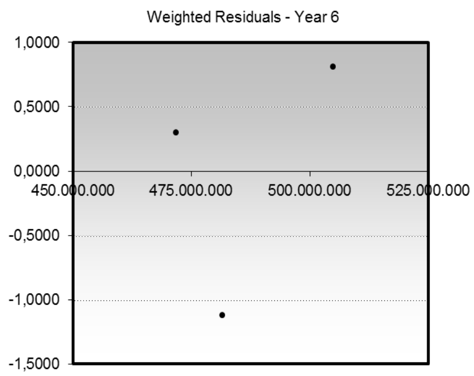


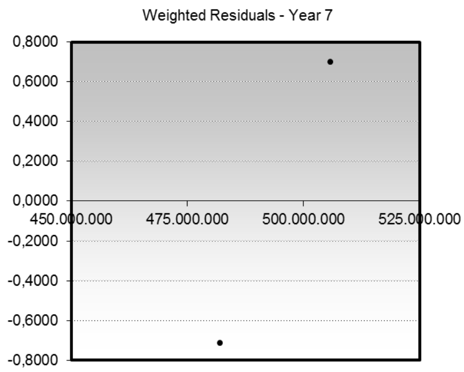




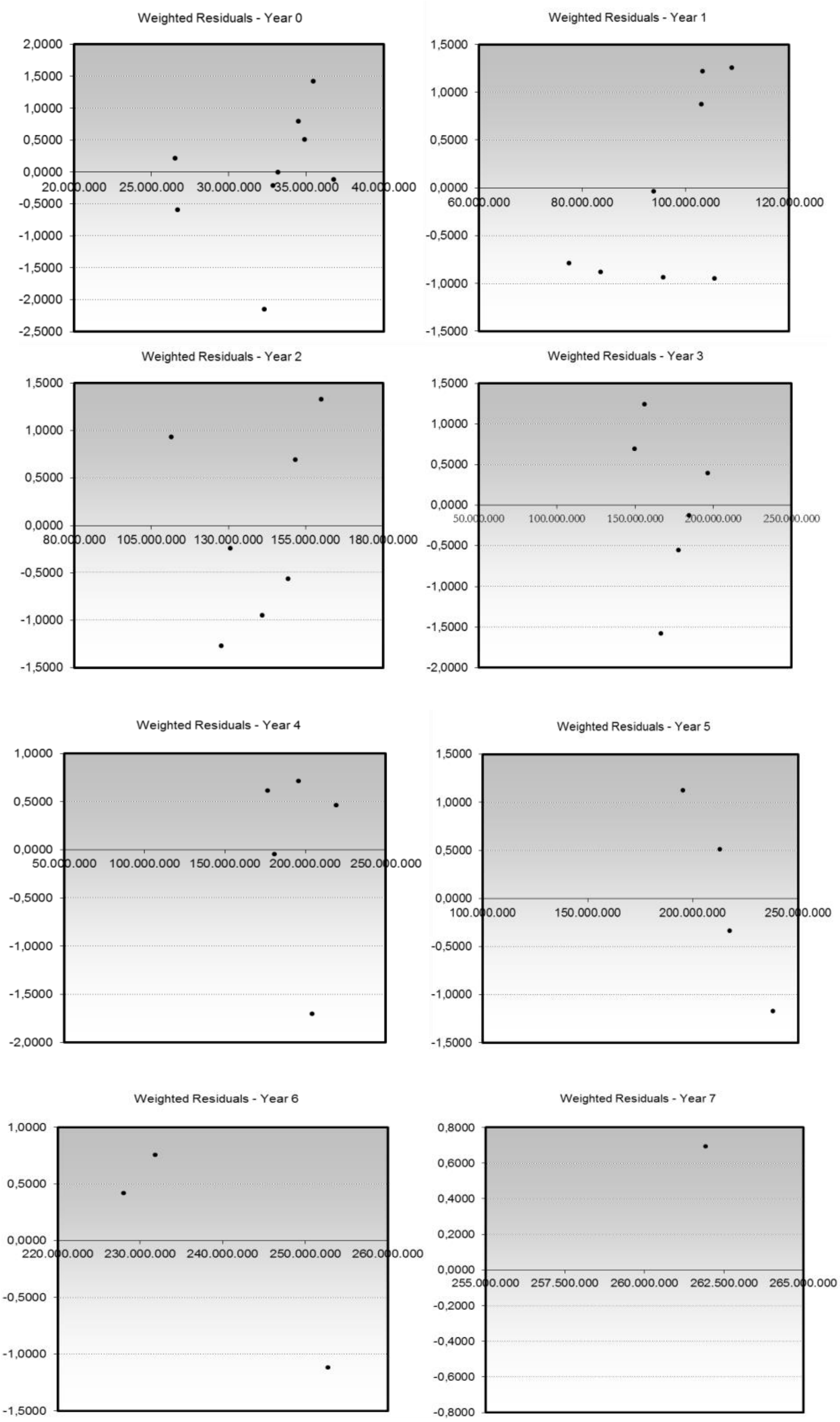








Bodily Injury



The graphs above show the differences between the observed data and the estimated data using the f_j parameter. Since none of the graphs indicates any trend, we can assume that the third assumption is verified for both HRG.

APPENDIX B - Over-Dispersed Poisson Generalized Linear Model Application

The constraint that the sum of incremental claims in each column is strictly positive is verified, since all incremental amounts in both run-off triangles are greater than 0. The parameter estimates as well as the respective standard errors for the GLM-ODP are presented in the tables bellow.

Material Damages

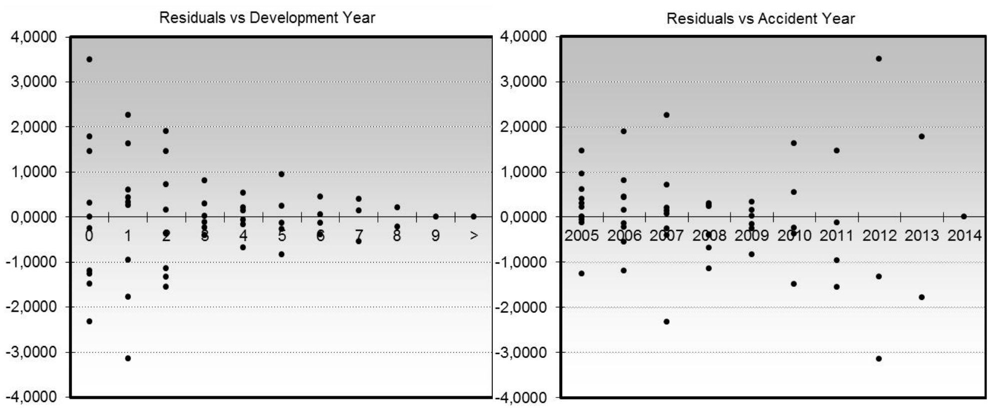
Parameters	Estimate	Standard Error
μ	19,6895	0,02405
α_1	-0,0485	0,03354
α_2	-0,0684	0,03373
α_3	-0,0634	0,0337
α_4	-0,0942	0,03399
α_5	-0,0372	0,03352
α_6	-0,0956	0,03409
α_7	-0,182	0,03499
α_8	-0,2091	0,03552
α_9	-0,1946	0,03909
β_1	-1,0218	0,01892
β_2	-3,4176	0,0575
β_3	-4,2875	0,09339
β_4	-4,9447	0,13924
β_5	-5,4226	0,19372
β_6	-5,9954	0,28676
β_7	-6,2758	0,37968
β_8	-6,1525	0,43405
β_9	-6,6721	0,7862

Bodily Injury

Parameters	Estimate	Standard Error
μ	17,5225	0,0452
α_1	-0,0836	0,048
α_2	-0,0984	0,0488
α_3	-0,1834	0,0507
α_4	-0,2072	0,0522
α_5	-0,2625	0,0545
α_6	-0,3664	0,0584
α_7	-0,4584	0,0646
α_8	-0,4534	0,0735
α_9	-0,375	0,113
β_1	0,6339	0,0396
β_2	0,1288	0,0452
β_3	-0,2076	0,0512
β_4	-0,6292	0,0613
β_5	-0,845	0,0708
β_6	-0,9331	0,0797
β_7	-1,3373	0,1061
β_8	-1,4686	0,1349
β_9	-1,9035	0,227
β_{10}	-1,074	0,1537

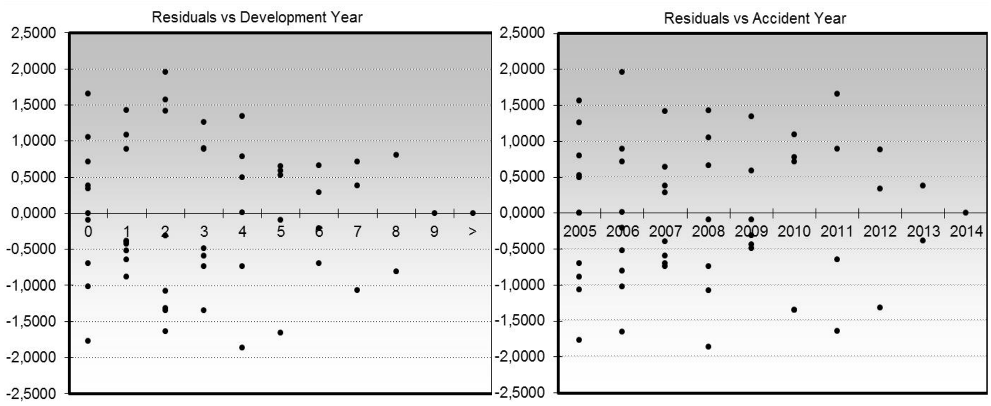
In order to analyze the quality of the adjustment provided by this model, the charts with the residuals are presented both for occurrence and development years. The standardized Pearson residuals were used for this purpose.

Material Damages



The first figure seems to show a decreasing trend, which seems to indicate that the fit to the MD data is not ideal.

Bodily Injury



Regarding the BI data set we verify, by analyzing the above graphs, that the residuals do not appear to present any systemic pattern.

Before proceeding with the bootstrap technique itself, it is necessary to check whether the residuals used satisfy the assumption of independence. Looking at the graphs above, the last three show a satisfactory (random) pattern, however the first one seems to show a decreasing trend. So it is not sufficiently clear that the independence assumption is verified for the MD data set.

APPENDIX C - EIOPA Euro Risk Free Rates

Maturity	Basic RFR	Basic RFR shock upwards	Basic RFR shock downwards
1	0,062%	1,062%	0,016%
2	0,075%	1,075%	0,026%
3	0,120%	1,120%	0,053%
4	0,184%	1,184%	0,092%
5	0,261%	1,261%	0,141%
6	0,343%	1,343%	0,199%
7	0,431%	1,431%	0,263%
8	0,529%	1,529%	0,339%
9	0,628%	1,628%	0,421%
10	0,723%	1,723%	0,499%
11	0,817%	1,817%	0,572%
12	0,895%	1,895%	0,635%
13	0,966%	1,966%	0,696%
14	1,018%	2,018%	0,733%
15	1,078%	2,078%	0,787%
16	1,119%	2,119%	0,806%
17	1,160%	2,160%	0,835%
18	1,200%	2,200%	0,864%
19	1,230%	2,230%	0,873%
20	1,265%	2,265%	0,898%
21	1,311%	2,311%	0,932%
22	1,364%	2,364%	0,972%
23	1,423%	2,423%	1,016%
24	1,485%	2,485%	1,062%
25	1,549%	2,549%	1,110%
26	1,613%	2,613%	1,158%
27	1,677%	2,677%	1,206%
28	1,740%	2,740%	1,253%
29	1,802%	2,802%	1,300%
30	1,863%	2,863%	1,347%
31	1,922%	2,922%	1,392%
32	1,979%	2,979%	1,436%
33	2,035%	3,035%	1,479%
34	2,088%	3,088%	1,520%
35	2,140%	3,140%	1,561%
36	2,190%	3,190%	1,600%
37	2,238%	3,238%	1,638%
38	2,284%	3,284%	1,674%
39	2,328%	3,328%	1,710%
40	2,371%	3,371%	1,744%

Source: EIOPA (December 2014).

APPENDIX D - Inflation Rates

Year	Inflation Rate
2005	2,3%
2006	3,1%
2007	2,5%
2008	2,6%
2009	-0,8%
2010	1,4%
2011	3,7%
2012	2,8%
2013	0,3%
2014	-0,3%
2015	0,5%

Source: Instituto Nacional de Estatística/PORDATA (2015).

APPENDIX E – Complementary Form of Method 1

The function π_t and the parameters $\hat{\delta}$ and $\hat{\gamma}$ referred in section 4.1.2 are presented below.

$$\pi_t(\hat{\delta},\hat{\gamma}) = \frac{1}{ln\left(1+\left(\left(1-\hat{\delta}\right)\cdot\frac{\bar{x}}{x_t}+\hat{\delta}\right)\cdot e^{2\cdot\hat{\gamma}}\right)}$$

where

$$\bar{x} = \frac{1}{T}\cdot\sum_{t=1}^{T} x_t$$

The parameters $\hat{\delta}$ and $\hat{\gamma}$ are obtaining by minimizing the following amount:

$$\sum_{t=1}^{T}\pi_t(\hat{\delta},\hat{\gamma})\left(\ln\left(\frac{y_t}{x_t}\right)+\frac{1}{2\cdot\pi_t(\hat{\delta},\hat{\gamma})}+\hat{\delta}-\ln(\hat{\sigma}(\hat{\delta},\hat{\gamma}))\right)^2-\sum_{t=1}^{T}\ln\left(\pi_t(\hat{\delta},\hat{\gamma})\right)$$

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